Institutions and Firms' Organization: Asymmetric Effects of Trade on Productivity and Welfare*

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Abstract

Contract enforcement and, more generally, the business environment play an important role in a world where producers of final goods need to source inputs from other suppliers. Weak institutions create uncertainty over the provision of intermediate goods demanded by final producers. Firms adapt the organization of their production to the local institutional environment. Compared to other models, we allow heterogeneous producers to choose their sector of production and we study how trade affects the relocation of final producers and resources across sectors. The quality of institutions and the ex-ante distribution of productivity determine the endogenous organization of firms and, in turn, the sector in which each final producer specializes. The best producers are shown to be relatively better at producing more complex goods and choose to specialize in the most complex sectors. We study how trade liberalization leads to asymmetric effects on the allocation of intermediate suppliers across final producers and across industries, as well as on aggregate productivity and welfare, when countries differ in institutional quality. Consistent with results in the literature, the model finds a positive effect of trade liberalization on aggregate productivity in the country with good institutions. On the other hand, it unveils a negative effect in the country with weak institutions. This asymmetric effect is larger when the difference in institutions is higher. In addition a large difference leads consumers from the country with good institutions that benefit from more varieties to lose in terms of purchasing power and aggregate utility.

Keywords: heterogeneous firms; firms organization; institutions; comparative advantage
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1 Introduction

The reallocation of resources across firms and sectors is a key factor for the economic development of a country. Theoretical papers such as Melitz (2003) and Bernard et al. (2007) and empirical studies such as Pavcnik (2002) and Trefler (2004) have shown that trade liberalization has a positive effect on aggregate productivity and it induces the reallocation of resources towards the most productive firms.[1] Some recent papers, however, provide evidence that these benefits depend on the existence of other non-trade distortions (see for example Freund and Bolaky (2008), Chang et al. (2009) and DeJong and Ripoll (2006)). These distortions, such low regulatory quality, financial constraints, or poor legal and political institutions, particularly affect developing countries and hamper their development.

In this paper, we develop a new channel that leads to distinctive results in terms of aggregate productivity and welfare. We propose a novel mechanism in which institutional distortions adversely affect the gains from trade. In particular the degree of difference in institutional quality between countries leads them to different specializations and creates asymmetric effects on productivity and welfare. This channel helps explaining how institutional distortions prevent countries, especially those with poor institutions, to benefit from the gains of trade described in the literature.

This paper focuses on differences in business-related institutions, such as contract enforcement, as an important source of comparative advantage (Levchenko (2007), Nunn (2007), Costinot (2009)). In our model, institutional obstacles to doing business affect the firms' choice of production, e.g. which good to produce and the organization of its production. At the country level, the quality of institutions affects how resources are allocated and used across sectors and therefore, at an international level, triggers the pattern of comparative advantage. In particular, countries with better institutions specialize in the production of more complex goods, while countries with weaker institutions specialize in simple industries.

Our theoretical framework delivers two key predictions on the effects of trade liberalization on aggregate productivity and welfare.

First, while it confirms a positive effect of trade on aggregate productivity in the country with good institutions, it unveils a negative effect in the country with weaker institutions, especially when the difference in institutions is very high and trade mainly happens across industries. This prediction results from the reallocation of resources triggered by both the specialization of a country and the endogenous production choices of firms. In fact, after liberalization, resources are reallocated from the comparative disadvantaged sector

[1] See also the detailed discussion that can be found in Harrison and Rodríguez-Clare (2010).

towards the comparative advantaged one. In addition, since the most productive firms always choose to produce the more complex good, in the country with good institutions resources are attracted by more productive firms and aggregate productivity goes up. The opposite happens in the country with weak institutions: the most productive firms, being in the comparative disadvantaged sector, release resources that are then absorbed by less productive firms. As a consequence of the expansion of the simple sector, new unproductive firms might even start producing. The country with weak institutions would thus see its resources be reallocated to the simple sector where less productive firms operate. This is part of the novel mechanism of our paper. Finally, the asymmetric effect on aggregate productivity is stronger and leads to a decline in aggregate productivity when the institutional difference between the countries, and thus the forces behind the reallocation of resources, are larger.

The second prediction has to do with how trade liberalization affects welfare through prices. In our model, a large difference in institutions is shown to increase the aggregate price and decrease consumers' welfare in the country with good institutions. The intuition is the following. In a monopolistic framework, consumers value diversity and consume all available goods. After trade liberalization, consumers from the country with good institutions have now access to and consume varieties produced in the other country. Since the other country has weaker institutions, the marginal costs of firms producing in this country are relatively higher and therefore their goods are relatively more expensive. In addition, when the gap in the quality of institutions between the trading partners is particularly high, the adverse effect of trade on prices and thus on welfare is amplified.

The new results of our paper are achieved thanks to the introduction of two novelties in the theoretical framework, namely the firm's organization that reflect how heterogeneous producers adapt to their local institutional environment and the endogenous choice of the sector by final producers. As to the first novelty, while relying on Costinot (2009) to model the firm's level impact of institutions on organization, we introduce firms heterogeneity and take into account the impact of institutions on aggregate productivity through the reallocation of resources. Firms optimally choose their horizontal degree of fragmentation by dividing the provision of their intermediate inputs among different suppliers.[2] The key trade-off comes from the gains and the costs of specialization. The gains are due to a fixed learning cost for each intermediate inputs to be supplied, and the costs from the probability that a supplier does not provide its subset of intermediate inputs. This probability ultimately depends on institutions in the form of contract enforcement. Better contract enforcement implies a higher probability that the supplier provides the intermediate inputs. This trade-off defines a marginal cost of production that depends on

[2]In a different set up, also Conconi et al. (2012) examine how trade liberalization affects the organizational structure of firms.

the productivity of each producer, the complexity of the good and the quality of contract enforcement.

Second, we build an original framework in which final producers endogenously choose their sector. Our approach differs from Bernard et al. (2007) where firms only decide whether to produce or not given the sector. In our model, producers choose their sector depending on their marginal cost of production and the aggregate prices. The marginal cost of production in a sector is a function of the idiosyncratic productivity of each producer and the quality of contract enforcement that determines its endogenous organization. Aggregate prices instead depend on the role of institutions in determining comparative advantage. In line with Costinot (2009) we show that the country with the best institutions has a comparative advantage in the complex industry whose outputs require a high number of intermediates. In this framework, the most productive firms are shown to always choose to produce the complex good for all level of contract enforcement. In contrast with Bernard et al. (2007) who find positive effects of trade on aggregate productivity for all possible cases, our model shows that introducing this endogenous choice might lead countries with weak institutions to lose in terms of productivity and welfare from trade liberalization.

The outline of the paper is as follows. In Section 2 we describe some stylized facts on the linkages between trade and productivity. Section 3 first details the equilibrium in autarky and the optimal organization of the firms. Then it studies the effect of trade openness with a focus on the free-trade case. Furthermore, we discuss here the extension of a costly trade equilibrium and show that it delivers similar qualitative results. Section 4 concludes.

2 Trade and productivity: new stylized facts

Some recent works have provided evidence that benefits from trade depend on the existence and the degree of other non-trade distortions and the feasibility of removing them. For example, Freund and Bolaky (2008) show that business regulation is an important complementary policy to trade liberalization. Their empirical analysis show that in countries with low barriers to entry there is a positive relationship between openness to trade and growth whereas in regulated economies the relationship is negative. Chang et al. (2009) provide evidence that, in addition to barriers to entry, also infrastructure development and labor market flexibility are crucial to enhance the growth effects of openness. [3] Our paper adds to this literature by constructing a framework in which business related institutions are crucial in the determination of gains from trade.

[3] DeJong and Ripoll (2006) find a positive relationship between tariffs and growth rates for the world's poorest countries, but a negative relationship for rich countries.

We explore how trade can affect economic performance and growth through its direct effect on productivity. Our model predicts that opening to trade can adversely affect the aggregate productivity in a country with weak institutions. Evidence of this negative effect of trade can be found in two recent papers and the case study illustrated below. Lu (2010) embeds the one-sector Melitz (2003) model into a comparative advantage framework and shows that in sectors where China has a comparative advantage, Chinese exporters were on average less productive than firms serving only the domestic market. Using Chinese data, Fan et al. (2011) show that the number of exporters and the share of exporting revenues are positively correlated with tariff in sectors with a comparative disadvantage.

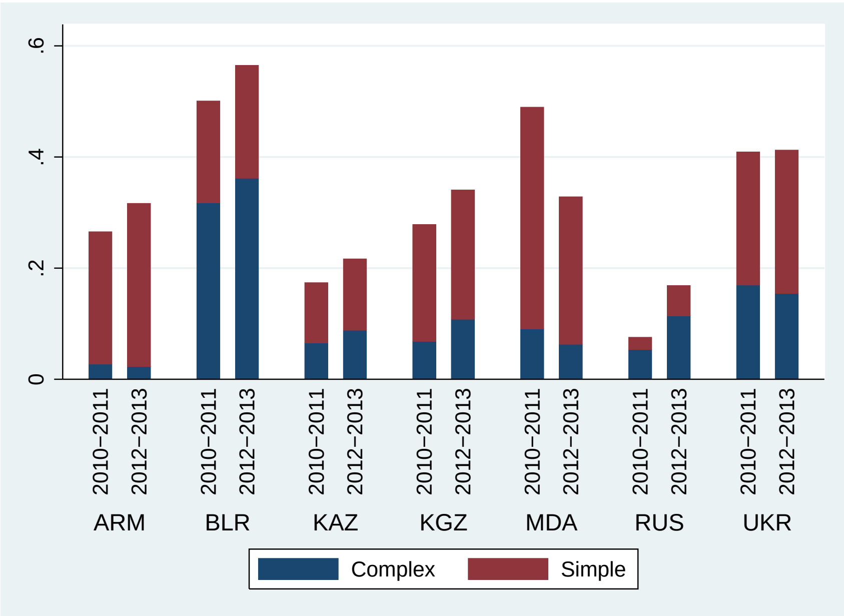
A recent liberalization episode among Commonwealth of Independent States (CIS) countries represents a good example of how institutional quality affects the gains from trade liberalization. The idea of a free trade area among CIS the emerged already right after the break up of the Soviet Union in 1991. Twenty years later, in October 2011, Russia, Ukraine, Belarus, Kazakhstan, Kyrgyzstan, Tajikistan, Moldova and Armenia signed a Treaty on a Free Trade Area between members of the Commonwealth of Independent States (CIS-FTA). The agreement was enforced starting from September 2012. The CIS-FTA simplified the network of trade relationship between CIS by replacing existing bilateral and multilateral trade agreements and effectively eliminated export and import duties on a host of goods.[4]

Export data from COMTRADE in figure 1 show that ex-Soviet countries are well integrated among each other: a part from Russia, between one fifth and more than half of the exports of CIS is directed towards other countries in the group. Moreover, figure 1 shows that intra-CIS exports increased for almost all countries in the period 2012-2013 after the entry into force of the CIS-FTA. The CIS-FTA thus represents a liberalization event that we can use to analyze the effects across industries of an increase in trade. Finally, the figure shows that countries like Armenia and Kyrgyzstan export mainly simple goods such as food and wearing apparel whereas Belarus and Russia export complex goods such as refined petroleum products and chemicals to other CIS countries.[5] The quality of institutions is a potential source of this pattern of specialization.

[4]Exemptions are included in the agreement but they will ultimately be phased out.

[5]Simple (complex) industries are industry with complexity below (above) the median. Details about the complexity of industries are reported in Appendix A.

Figure 1: Average exports between CIS, 2010-2013



The historical experience and data from the World Bank suggest that business-friendly institutions are likely to be an important issue in CIS. The Doing Business database provides information about the quality of business related institutions for all countries in the World. Table 1 shows the quality of contract enforcement in the countries involved in the CIS-FTA.[6] Among this sample of countries, Belarus has the best contract enforcement whereas Armenia lacks behind all other CIS.[7]

Measures of productivity for Armenia, Belarus, Kazakhstan, Kyrgyzstan, Moldova, Russia and Ukraine at the industry level (2 digits ISIC Rev. 3.) before and after the CIS-FTA can be constructed using the firm level data available in the World Bank Enterprise Survey. Details about the dataset and the construction of productivity are provided in Appendix A. We can then determine if changes in exports or comparative advantage are positively related to changes in productivity in these countries during a liberalization episode.

Armenia and Kyrgyzstan, the countries with the lowest level of contract enforcement among CIS, experienced a decrease in average aggregate productivity after 2012.[8] Moreover, a more disaggregated analysis shows that, in the period under consideration, Armenia experienced an increase in revealed comparative advantage in manufacturing of food

[6]As defined in the dataset, contract enforcement assesses the efficiency of the judicial system by following the evolution of a commercial sale dispute over the quality of goods and tracking the time, cost and number of procedures involved from the moment the plaintiff files the lawsuit until payment is received. For additional details, see the Doing Business web page http://www.doingbusiness.org/

[7]The average and median levels of contract enforcement in the World in the period 2010-2013 are 60 and 60.4 respectively. The variance of the variable is 164.1 in the sample of all countries, and 55.9 in the CIS sample.

[8]In our data, also Moldova, Russia and Belarus present lower aggregate productivity in 2012 and 2013 with respect to 2008 and 2009 while Ukraine and Kazakhstan have higher aggregate productivity.

Table 1: Average contract enforcement in CIS, 2010-2013

Country	AVG contract enforcement DTF
Armenia	55.35
Belarus	79.90
Kazakhstan	68.02
Kyrgyzstan	64.63
Moldova	74.78
Russia	76.11
Tajikistan	67.76
Ukraine	67.19

Note: Averages over the period 2010-2013 of distances to the frontier of contract enforcement are reported. Higher values correspond to better institutions.

and beverages, a simple industry, but the average productivity in that industry decreased sharply.[9] A negative relationships between improvements in comparative advantage and declines in productivity can be found in manufacturing of textiles, another simple sector, in Kyrgyzstan. In Ukraine too, increases in comparative advantage in manufacturing of food and beverages and non-metallic mineral products have been accompanied by decreases in productivity. [10]

The examples of Armenia and Kyrgyzstan reported above are not definitive evidence of negative effects of trade in countries with weak institutions and we are not claiming any causal relationship. However, this simple empirical evidence suggests that the positive selection of firms triggered by trade liberalization is complex and depends on additional factors such as the quality of institutions.

[9] Revealed comparative advantage is calculated using the Balassa index, Balassa (1965).

[10] A weak negative correlation between changes in RCA and changes in TFP in countries with weak institutions can also be found in a wider sample of countries. We also run a simple OLS regression using data from all countries surveyed from the World Bank. Controlling for country-industry variables such as the share of imports of an industry in a country and the country share of world imports in an industry, time-, country-, and industry-fixed effects, the correlation between changes in RCA and changes in TFP is positive but not significant. However, the coefficient of an interaction term between changes in RCA and a dummy equal to one for weak institutions suggests that there is a negative significant correlation between the two variables in countries with weak institutions.

3 The model

3.1 The economic environment

We consider two countries indexed by $k \in \{H, F\}$ that have similar economic structures. Each country has two sectors, S and A, producing differentiated goods under monopolistic competition and a numeraire sector, X, producing a homogenous good under perfect competition[11]. S and A produce respectively simple and advanced goods. The production of simple goods is characterized by a lower degree of complexity (properly defined later). Each country has a population of L workers and there is no mobility of workers across countries. Every worker is endowed with a fixed number of hours h. We first describe in detail the economic structure in country H.

3.1.1 Demand

We assume Cobb-Douglas utility across sectors and CES across varieties:

$$U = \mathbf{S}^{\alpha_S}\mathbf{A}^{\alpha_A}X^{\alpha_X}$$

where $\mathbf{S}$ and $\mathbf{A}$ are the standard aggregate consumption levels for simple and advanced goods defined as

$$\mathbf{S} := \left[\int_{\omega\in\Omega^S} c(\omega)^{\frac{\sigma-1}{\sigma}}\mathrm{d}\omega\right]^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad \mathbf{A} := \left[\int_{\omega\in\Omega^A} c(\omega)^{\frac{\sigma-1}{\sigma}}\mathrm{d}\omega\right]^{\frac{\sigma}{\sigma-1}} \quad \text{with } \sigma > 1.$$

Ω^i stands for the set of available varieties for each sector with $i \in \{S, A\}$. We assume $\alpha_X, \alpha_S, \alpha_A > 0$ and $\alpha_X + \alpha_S + \alpha_A = 1$.

3.1.2 Supply: Final Firms and Suppliers

Simple and advanced goods have to be produced according to their degree of complexity, which is the size of the continuum of intermediate goods required for the final production. The production of a simple good requires fewer intermediate goods than the production of an advanced good. We denote by z^i the size of this continuum for $i \in \{S, A\}$, with $z^S < z^A$. For the sake of clarity we explicitly distinguish between final and intermediate goods, the

[11]The presence of the numeraire allows us to pin down the wage level and to focus on the price effects of trade liberalisation. The homogeneous numeraire good is produced under perfect competition. One unit of X requires one unit of labor to be produced, so that the wage in the numeraire sector is $w = 1$. At the equilibrium, within country labor mobility makes sure that the wage w_i is the same for the sectors $i \in \{S, A\}$. For the rest of the paper we denote w the wage for all the sectors and we will focus our discussion on the the two sectors S and A.

former being the ones entering the consumption bundle. Moreover, we call *final firms* (or simply *firms*) the producers of the simple and advanced final goods. Intermediate goods instead are provided by *suppliers* (properly defined later).

For each sector, the problem of a final firm is to efficiently organize the production of all the intermediate goods across suppliers. We assume that a final firm is characterized by an exogenous, idiosyncratic level of productivity φ. The productivity of the final firm affects the productivity of its suppliers as well as the way suppliers are organized to produce the final good[12]. The parameter φ is distributed according to a probability density function g on the support $(0, +\infty)$. We denote with G the associated cumulative distribution function. We posit that g is the same for the two countries. Given productivity φ, a final firm will choose whether to produce and in which sector to do so. Contrary to most of the models with multi-sectors economies and a monopolistic competition (e.g. Bernard et al. (2007)), in our framework the final firms choose in which sectors to produce and are not ex-ante affiliated to one sector.

For simplicity, we assume that one supplier consists of one worker endowed with h working hours. For each intermediate good, the supplier has to first spend time learning how to produce it. Then, actual production happens through a linear technology. The productivity of a supplier depends on the productivity of the final firm. Consider a supplier that has to provide a certain number of intermediate goods for a final firm with a productivity φ. For each intermediate good the supplier needs $\frac{1}{\varphi}$ hours to learn how to produce it and $\frac{1}{\varphi}$ hours for the actual production of one unit of it. The higher the productivity of the final firm, the more productive to learn and to produce a supplier becomes.

Denote with $\mathbb{Y}(\varphi)$ the number of final good's units u that a final firm with productivity φ plans to produce. The number of hours l necessary to learn and produce one intermediate good for the production of $\mathbb{Y}(\varphi)$ units of the final variety are given by the following expression:

$$l := \int_{u \in \mathbb{Y}(\varphi)} \frac{1}{\varphi} \mathrm{d}u + \frac{1}{\varphi} \tag{3.1}$$

The learning cost of one intermediate good and the marginal productivity of a supplier in a final firm with productivity φ are the same across sectors.

Final firms produce under monopolistic competition and face a fixed production cost $f > 0$. We assume that all the sector-specific intermediate goods have to be provided in order to produce one unit of any final variety[13].

[12] We can consider this productivity level as a final firm-specific knowledge or as the ability of its manager.

[13] This is analogous to the O-ring theory by Kremer (1993).

3.1.3 Firms' Organization and Institutions

Our modeling strategy for the organization of the final firms follows closely the theoretical structure introduced by Costinot (2009).

Let us consider a final firm with productivity φ in sector i. Each unit of the final good that the firm wants to produce requires one unit of each intermediate good in $[0, z^i]$. The final firm has to choose the number of its suppliers - we posit that suppliers cannot produce intermediates for more than one final firm - and, most importantly, it has to allocate the provision of intermediate goods across them. The final firm pays a wage w to each chosen supplier, irrespectively of the actual provision of the intermediate goods. It can be shown that the final firm optimally partitions the interval $[0, z^i]$ into N identical ranges of intermediate goods and assigns each range to a different supplier. Moreover, it optimally assigns the same range to the same supplier across as many units of final goods as it takes to deplete the supplier's endowment of hours[14]. As a result, the suppliers chosen by the final firm are divided into groups of size N. Each member of a group is specialised in z^i/N intermediate goods: it spends $z^i/N\varphi$ hours in learning how to produce them, and the remaining $h - z^i/N\varphi$ hours of its endowment in producing them.

We crucially assume that the suppliers' activity can be hampered by institutional obstacles such as corrupted bureaucracies, unexpected taxation or violation of property rights[15]. The quality of institutions, therefore, determines the probability with which every single supplier is able to fulfill the provision of intermediates it has been assigned to. Formally, we define a successful provision indicator for a given supplier as

$$\mathbb{I}(supply) = \begin{cases} 1 & \text{with probability } e^{-\frac{1}{\theta}} \\ 0 & \text{with probability } 1 - e^{-\frac{1}{\theta}} \end{cases} \tag{3.2}$$

where $\theta > 0$ captures the quality of institutions. When $\mathbb{I}(supply) = 0$ the supplier fails the provision of all the intermediate goods it was responsible for. As a consequence, the final firm is not able to produce those units of the final good, which the supplier's provision was intended to contribute to. Low values of θ are associated with low probabilities of successful provision and therefore represent weak institutional frameworks. For θ going to $+\infty$ instead, the probability of successful provision tends to 1, minimizing the uncertainty in the production process of the final firm.

The optimal organization of a final firm coincides with the optimal choice of N, the size of the suppliers' group producing intermediates for each unit of final good or, in

[14] Our framework takes as given many important intermediate results of the Cosinot theoretical structure. We provide a fully micro funded application in Appendix B.

[15] A complementary assumption would be the existence of imperfect contract enforcement. In this environment a supplier is able, with a certain probability, to shirk on the provision of intermediates that was assigned to it by a final firm.

other words, the degree of fragmentation of intermediates' provision across suppliers. The trade-off behind this optimal decision is intuitive: on the one hand, a higher fragmentation allows the final firm to leave its suppliers with a greater amount of hours for the actual production of intermediates (each supplier is specialized in a smaller range of intermediates and therefore has to allocate less hours into learning). On the other hand, a higher degree of fragmentation enhances uncertainty in the production process of the final firm: a single supplier failing its provision compromises the production of units of final goods, independently on the provision of all the other members of its group.

In our model, institutions affect the organization of the final firms and their frontier of production. Moreover, the quality of institutions is the only parameter that differs across the two countries. If the two countries trade among each other, institutional heterogeneity is the source of comparative advantage and therefore it creates potential trade opportunities. Before turning to the analysis of trade regimes, we present our modelling framework and derive results for a country in autarky.

3.2 Equilibrium under autarky

3.2.1 The consumers' problem

We apply the two-stage budget procedure using the aggregate income $\mathcal{R}$ and the aggregate price indexes

$$P^i = \left[\int_{\omega\in\Omega^i} p(\omega)^{1-\sigma}\mathrm{d}\omega\right]^{\frac{1}{1-\sigma}} \qquad \forall i \in \{S, A\}$$

The Cobb-Douglas specification implies fixed expenditure shares for the two sectors: $P^S\mathbf{S} = \alpha_S\mathcal{R}$ and $P^A\mathbf{A} = \alpha_A\mathcal{R}$. In order to get rid of any demand side effects in determining the comparative advantage under free trade we assume $\alpha_S = \alpha_A = (1-\alpha_X)/2$.[16] We denote by α this parameter. In addition we take R as the aggregate income net of the expenditure for the numeraire good X, $R = (1-\alpha)\mathcal{R}$. For every sector, consumption across varieties is given by the following equations:

$$c(\omega) = \begin{cases} \mathbf{S}\big[p(\omega)/P^S\big]^{-\sigma} & \text{if } \omega \in \Omega^S \\ \mathbf{A}\big[p(\omega)/P^A\big]^{-\sigma} & \text{if } \omega \in \Omega^A \end{cases} \tag{3.3}$$

3.2.2 The firms' problem: optimal organization

The final firm chooses how to organize its production through the allocation of the intermediate-good production among the suppliers. The optimal organization strategy

[16]Krugman (1980) shows how the country with higher internal demand for a sector will develop a comparative advantage in the production of the sector specific varieties.

is a number of suppliers denoted by N (called degree of fragmentation) associated to an optimal allocation of intermediate goods for each supplier.

First, we denote by $y(\varphi)$ the expected production given the initial plan of production $\mathbb{Y}(\varphi)$ that is produced in case of no uncertainty. Given that all suppliers have the same probability to fail intermediates' provision, the expected production of the final firm is given by:

$$y(\varphi) = \mathbb{P}(\mathbb{I} = 1)^{N(\varphi)} \int_{u \in \mathbb{Y}(\varphi)} du \tag{3.4}$$

with $N(\varphi)$ the number of suppliers in a team of a final firm with productivity φ. $\mathbb{P}(\mathbb{I} = 1)^{N(\varphi)}$ defines the probability that all the suppliers successfully provide their range of intermediate goods such that the final good can be produced. Supplier level probabilities of failed provision are multiplied by each other because the final good is produced only if all the intermediate goods required to its production are supplied.

We can derive the production technology of a final firm of productivity φ in the sector with complexity $z \in \{z^S, z^A\}$ of a country with institutions θ and determine its optimal organization $N^*(\varphi, z, \theta)$. Given the total mass S of suppliers working in in the final firm, its maximization problem can be written as[17]

$$\max_N \; pe^{-\frac{N}{\theta}} \frac{\varphi}{z^i} S\left(h - \frac{z^i}{\varphi N}\right) - w(S + f) \tag{3.5}$$

The optimal organization - or degree of fragmentation - of the final firm is given in the following

Proposition 1 *(Degree of fragmentation) The optimal number of suppliers for a final firm with productivity φ in the sector with complexity z in a country with institutions θ is:*

$$N^*(\varphi, z, \theta) = \frac{z}{2h\varphi}\left(1 + \sqrt{1 + \frac{4\theta h \varphi}{z}}\right) \tag{3.6}$$

Proof. See Costinot (2009).

The final good is produced when each of the N suppliers have supplied their range of intermediate goods. The degree of fragmentation depends upon exogenous parameters as stated in the following

Observation 1 *(Comparative statics) N^* decreases in φ, increases in z and θ.*

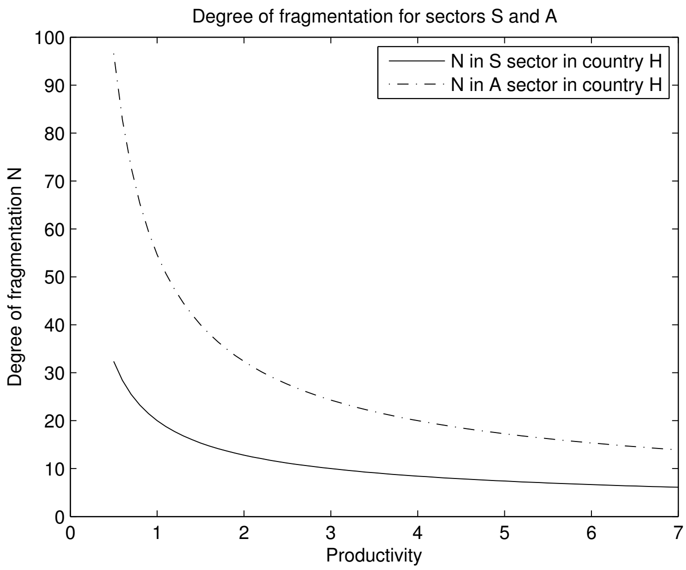
This comparative static result tells us that higher productivity, lower complexity or worse

[17] The computation is similar to Costinot (2009) and is detailed in the Annex. $e^{-\frac{N}{\theta}}$ is the probability for teams of N suppliers to get all the intermediate goods provided and $\left(h - \frac{z^i}{N\varphi}\right)$ the number of hours left for production for each supplier after the learning process.

institutions decreases the fragmentation of the production by the final firm. This comes from the trade-off explained in Costinot (2009) between the gains and costs of fragmentation. The learning cost for each intermediate good creates gains of fragmentation as a supplier with a smaller interval of goods can be more specialized and produce more. However the uncertainty in the supply of intermediates due to the poor quality of institutions creates costs of fragmentation of the final production.

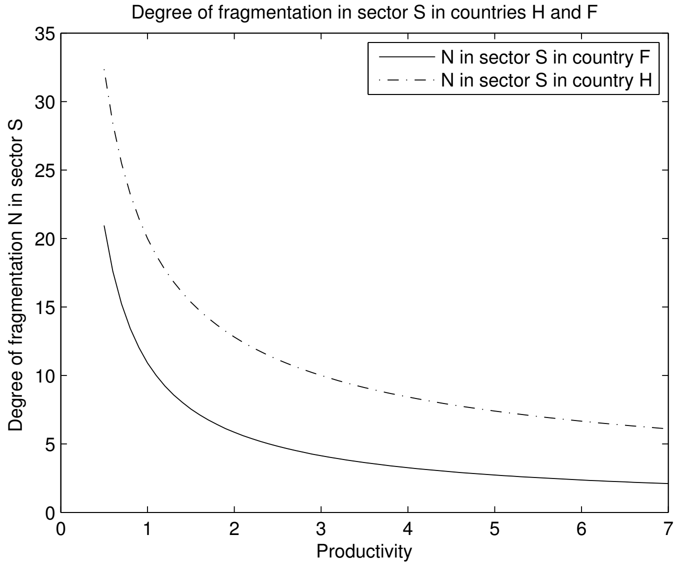
A higher productivity decreases the learning cost per supplier but does not affect the uncertainty level due to the quality of institutions. The gains of fragmentation are reduced with a higher productivity and the final firm decreases its optimal degree of fragmentation. Second, a higher degree of complexity for the final good increases the number of intermediate goods to provide and the hours to be dedicated to the learning process. The gains of fragmentation increase with a higher degree of complexity and the final firm expands its optimal degree of fragmentation. Finally, a higher quality of institutions directly decreases the costs of fragmentation and the final firm increases its optimal degree of fragmentation. We provide a graphical illustration of the comparative statics result in Figure 2 and 3.[18]

Figure 2: The degree of fragmentation N^* for the two sectors S and A in one country



[18]The general patterns shown in Figure 2 hold for any level of institutions. The general patterns in figure 3 hold for any level of complexity.

Figure 3: The degree of fragmentation N^* for two countries with different qualities of institutions θ^H and θ^F in one sector



The fragmentation of production directly affects production chains, outsourcing and the productivity of firms. One example is Fally (2012) that shows that fragmentation weighted by the value added of each range of intermediates has decreased over the last decades in the US. The explanation he gives is the increase of services in production that are usually not so fragmented and are provided close to the customers. Our model provides another mechanism for which a higher productivity of final firms, a lower complexity of final goods or a fall in the quality of institutions can also explain this fall of fragmentation.

3.2.3 The firms' problem: production and sector decision

In this subsection we derive the optimal pricing rule and the profit function for firms of productivity φ. We then determine which firms choose to produce and in which sector they do so. For the rest of the paper we denote by $N^i(\varphi)$ the optimal organization of the final firm of productivity φ in sector $i \in \{S, A\}$ in a country with a quality of institutions θ, such that $N^i(\varphi) = N^*(\varphi, z^i, \theta)$.

Let us consider a final firm with a productivity level φ producing a variety in Ω^i under the institutional framework θ. The final firm chooses the optimal total mass of suppliers $S^i(y)$ summing up all the suppliers required to produce y, the whole amount of final good:

$$S^i(y) = \frac{z^i}{\varphi} e^{\frac{N^i(\varphi)}{\theta}} \left(h - \frac{z^i}{\varphi N^i(\varphi)}\right)^{-1} y \tag{3.7}$$

Given optimal organization, we define the inverse of the marginal productivity of a final firm's supplier as[19]

$$\beta^i(\varphi) := \frac{\partial S^i(y)}{\partial y} = e^{\frac{N^i(\varphi)}{\theta}}\left[\frac{h\varphi}{z^i} - \frac{1}{N^i(\varphi)}\right]^{-1} \tag{3.8}$$

The maximization problem of the final firm can be written as

$$\max_y \; p^i(y)y - w\Big[S^i(y) + f\Big] \tag{3.9}$$

For the rest of the paper we set the wage w equal to 1. Following Dixit and Stiglitz (1977) we posit that the market share of each final firm is small enough in order to be neglected in the pricing decision of the others. This assumption (supported by the infinite number of firms in our set up) together with the constant elasticity of substitution gives us the following expression for the elasticity of demand faced by the final firm:

$$\epsilon^i(\varphi) = \epsilon = \frac{1}{1-\rho} \quad \text{where} \quad \rho = \frac{\sigma - 1}{\sigma} \tag{3.10}$$

The pricing rule is defined by the standard mark-up over the marginal cost:

$$p^i(\varphi) = \frac{\beta^i(\varphi)}{\rho} \tag{3.11}$$

The profit function is given by

$$\pi^i(\varphi) = \frac{R}{2\sigma}\left[\frac{P^i\rho}{\beta^i(\varphi)}\right]^{\sigma-1} - f \tag{3.12}$$

Let us begin our analysis of the profit function with the following

Observation 2 *(Properties of the profit function)* $\forall\varphi, \forall i$ $\pi^i(\varphi)$ *is continuous and monotonically increasing in* φ. *Moreover* $\lim_{\varphi\to 0}\pi^i(\varphi) = -f$ *and* $\lim_{\varphi\to+\infty}\pi^i(\varphi) = +\infty$.

The contribution of this paper is to allow final firms to be mobile across sectors. Each final firm optimally chooses in which sector to produce depending on the expected profits in each sector given its productivity. Optimal production and sector decision under autarky is given by the following

Proposition 2 *(Production and sector decision) If the autarky equilibrium (properly defined later) exists, (i) there exists one productivity threshold* φ^{SA} *such that* $\pi^S(\varphi^{SA}) = \pi^A(\varphi^{SA}) > 0$; *(ii) there exist two productivity thresholds* φ^{eS} *and* φ^{eA} *such that* $\pi^S(\varphi^{eS}) = \pi^A(\varphi^{eA}) = 0$

[19] This level of productivity differs from the initial distribution of productivity parameters φ and results form the optimal strategy of the firm to organize the production depending on the complexity of the goods.

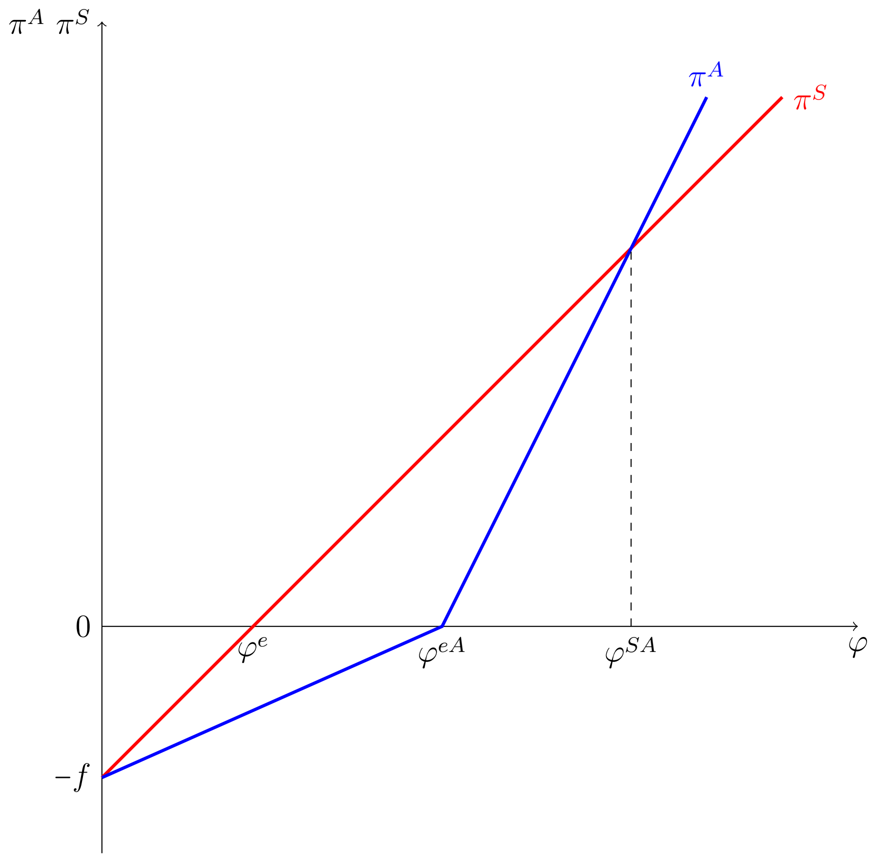
and $\varphi^{eS} < \varphi^{eA}$; (iii) a final firm chooses whether and in which sector to produce according to the following scheme:

- *if $\varphi < \varphi^{e}$ with $\varphi^{e} = \varphi^{eS}$, the firm does not produce any good,*
- *if $\varphi \in [\varphi^{e}, \varphi^{SA})$, the firm produces a variety in sector S,*
- *if $\varphi \geq \varphi^{SA}$, the firm produces a variety in sector A.*

Proof. See Appendix C.

Proposition 2 shows the existence of the two thresholds φ^{eS} and φ^{eA} from which a firm can make non negative profits. The threshold φ^{eS} is shown to be the lowest level of productivity that enables a firm to make non negative profits, we call it the *entry threshold* and we drop the S from its superscript. A firm that draws a productivity parameter below φ^{e} exits the market and never starts producing. The choice threshold φ^{SA} is defined as the productivity level for which a firm is indifferent between producing in one of the two sectors. We provide a graphical representation of the entry and choice thresholds in Figure 4 where we rely on a simplified representation of the profit functions for the two sectors.

Figure 4: Profits as function of productvity



Proposition 2 also states that for any quality of institutions, firms in the advanced sector are more productive than the firms in the simple sector. A firm with a productivity between φ^e and φ^{SA} produces a simple variety, and with a productivity above φ^{SA} an advanced variety. This important result is explained by the fact that the ratio of the marginal costs $\beta^S(\varphi)/\beta^A(\varphi)$ is increasing in the productivity. This implies that final firms are increasingly better at producing a variety in sector A relatively to a variety in sector S. What matters here is the relative ratio, as more productive firms are always better (lower marginal costs) to produce a variety in each sector. However more productive firms are relatively better at producing a variety in sector A.

3.2.4 Aggregation: prices and profits

We define the average marginal costs $\tilde{\beta}^S$ and $\tilde{\beta}^A$ in the two sectors which is determined by the cutoff productivity levels φ^e and φ^{SA} as follows.

$$\tilde{\beta}^S = \tilde{\beta}^S(\varphi^e, \varphi^{SA}) = \left[\frac{1}{G(\varphi^{SA}) - G(\varphi^e)} \int_{\varphi^e}^{\varphi^{SA}} \left(\beta^S(\varphi)\right)^{1-\sigma} g(\varphi)\mathrm{d}\varphi\right]^{\frac{1}{1-\sigma}}$$

and

$$\tilde{\beta}^A = \tilde{\beta}^A(\varphi^{SA}) = \left[\frac{1}{1 - G(\varphi^{SA})} \int_{\varphi^{SA}}^{\infty} \left(\beta^A(\varphi)\right)^{1-\sigma} g(\varphi)\mathrm{d}\varphi\right]^{\frac{1}{1-\sigma}}$$

Calling M the total mass of firms active either in S or in A, we can write the aggregate price indexes for the two sectors as

$$P^S = \left(M^S\right)^{\frac{1}{1-\sigma}} p^S\left(\tilde{\beta}^S\right) \quad \text{and} \quad P^A = \left(M^A\right)^{\frac{1}{1-\sigma}} p^A\left(\tilde{\beta}^A\right).$$

with $M^S = \left[\frac{G(\varphi^{SA})-G(\varphi^e)}{1-G(\varphi^e)}\right]M$ and $M^A = \left[\frac{1-G(\varphi^{SA})}{1-G(\varphi^e)}\right]M$, denoting respectively the mass of firms producing a variety of the simple and the advanced goods. Finally, aggregate profits Π are given by the following expression:

$$\Pi = M\bar{\pi} = M\left[\frac{G(\varphi^{SA}) - G(\varphi^e)}{1 - G(\varphi^e)}\bar{\pi}^S + \frac{1 - G(\varphi^{SA})}{1 - G(\varphi^e)}\bar{\pi}^A\right]$$

with $\bar{\pi}^S$ and $\bar{\pi}^A$ the average profits defined as

$$\bar{\pi}^S = \frac{\int_{\varphi^e}^{\varphi^{SA}} \pi^S(\varphi)g(\varphi)\mathrm{d}(\varphi)}{[G(\varphi^{SA}) - G(\varphi^e)]} \quad \text{and} \quad \bar{\pi}^A = \frac{\int_{\varphi^{SA}}^{\infty} \pi^A(\varphi)g(\varphi)\mathrm{d}(\varphi)}{[1 - G(\varphi^{SA})]}$$

3.2.5 Timing and free-entry condition

Following Melitz (2003) we model a process of firms' dynamics. Every period there is a mass M_e of potential entrants. At this stage the potential entrants are identical. In order to draw a productivity parameter from the distribution $g(\cdot)$ they have to pay a fixed entry cost f_e thereafter sunk. Once the firm knows its productivity, it decides whether to engage in production and in which sector to do so. Those decisions are taken anticipating optimal pricing behavior, which in turn embeds optimal organization determined taking prices as given.[20] Thus, only the potential new firms with a productivity level higher than φ^e finally enter the production process. Every period will be characterized by a mass M of active firms which is the sum of the firms active in the two sectors: $M = M^A + M^S$. For every active firm in every period, there is a positive probability δ of exogenous death. At the beginning of the period a proportion δ of the incumbent firms M_{-1} disappears. The dynamics is given by: $M = (1-\delta)M_{-1} + (1-G(\varphi^e))M_e$. We will focus on the steady states of this dynamic process, where $M = M_{-1}$ and $[1 - G(\varphi^e)]M_e = \delta M$. The expected profits from drawing a productivity level has to be equal to the cost f_e of having a draw. From this we derive the firm entry condition:

$$V = \frac{[1 - G(\varphi^e)]}{\delta}\bar{\pi} = f_e \tag{3.13}$$

with V the ex-ante utility of the firm over time and $\bar{\pi}$ the average ex-post profit in the economy. We use the expressions of the average profits to rewrite the free-entry condition as a function of the the two thresholds (φ^e and φ^{SA}) and other exogenous variables:

$$V(\varphi^e, \varphi^{SA}) = \tfrac{f}{\delta}\Big\{[G(\varphi^{SA}) - G(\varphi^e)]\Big\{\Big[\tfrac{\tilde{\beta}^S(\varphi^e,\varphi^{SA})}{\beta^S(\varphi^e)}\Big]^{1-\sigma} - 1\Big\} +$$
$$+[1 - G(\varphi^{SA})]\Big\{\Big[\tfrac{\tilde{\beta}^A(\varphi^{SA})}{\beta^A(\varphi^{SA})}\tfrac{\beta^S(\varphi^{SA})}{\beta^S(\varphi^e)}\Big]^{1-\sigma} - 1\Big\}\Big\} = f_e$$

3.2.6 Goods and labor markets

The goods market clearing condition requires that the share of revenues from a sector equals the share of expenditures into it:

$$R_S = \alpha_S R \quad \text{and} \quad R_A = \alpha_A R$$

Suppliers are used to enter the production process as well as to produce. S^e denotes the total number of suppliers used in the entry process (notice that S^e is not sector specific) and S^p_i denotes the number of suppliers used for production in sector i. Given our simplifying assumption of one worker for each supplier, the total number of suppliers is equal to the number of workers L. The labor market clearing conditions is thus:

$$S^e + S^p = L \quad \text{with} \quad S^p = S^p_S + S^p_A$$

[20]As in Dixit and Stiglitz (1977) we assume that the market shares of the firms are small enough not to trigger the strategic consideration of the opponents' pricing behavior.

3.2.7 Equilibrium

Proposition 3 *(Autarky equilibrium) For each country, there exists an autarky equilibrium*

$$\{\varphi^{e*}, \varphi^{SA*}, P^{S*}, P^{A*}, M^*, p^{S*}(\varphi), p^{A*}(\varphi)\}$$

that verifies the optimal behaviour of the consumers and producers, the labor market and good market conditions.

Proof. See Appendix C.

All the equilibrium endogenous variables can be pinned down from the vector of thresholds $(\varphi^{e*}, \varphi^{SA*})$. See Appendix C (Proof of Proposition 3) for a detailed derivation of the equilibrium under autarky.

Observation 3 *(Institutions under autarky) Under the autarky equilibrium, (i) the entry and choice thresholds φ^{e*} and φ^{SA*} decrease in the quality of institutions; (ii) the marginal costs at both thresholds $\beta^S(\varphi^{e*})$ and $\beta^A(\varphi^{SA*})$ decrease in the quality of institutions; (iii) the average numbers of suppliers per team $\tilde{N}^S$ and $\tilde{N}^A$, i.e. the average degrees of fragmentation, decrease in the quality of institutions.*

Better institutions decrease the cost of production by reducing the uncertainty with which suppliers provide their range of intermediate goods. As a consequence, better institutions reduce the marginal production cost and allow firms with a low exogenous productivity to start producing (entry threshold decreasing in θ). A change in θ affects also the marginal cost $\beta^S(\cdot)$. Following an increase in the quality of institutions, the worst producing firm has a lower exogenous productivity but also a lower marginal cost. The same happens for the worst firm producing in the advanced sector. Finally, we define the average degree of fragmentation in the two sectors by:

$$\tilde{N}^S = \tilde{N}^S(\varphi^e, \varphi^{SA}) = \left[\frac{1}{G(\varphi^{SA}) - G(\varphi^e)} \int_{\varphi^e}^{\varphi^{SA}} \left(N^S(\varphi)\right)^{1-\sigma} g(\varphi)\mathrm{d}\varphi\right]^{\frac{1}{1-\sigma}}$$

and

$$\tilde{N}^A = \tilde{N}^A(\varphi^{SA}) = \left[\frac{1}{1 - G(\varphi^{SA})} \int_{\varphi^{SA}}^{\infty} \left(N^A(\varphi)\right)^{1-\sigma} g(\varphi)\mathrm{d}\varphi\right]^{\frac{1}{1-\sigma}}$$

The average degree of fragmentation in both sectors increase in the quality of institutions. A lower uncertainty about the provision of the intermediate goods leads to higher equilibrium gains of fragmentation.

Figures 5, 6 and 7 provide a graphical representation of Observation 3 using the results from a numerical simulation of the equilibrium under autarky[21]. The figures plot equilib-

[21] The parametrisation of our economic framework follows closely the numerical exercise in Bernard

rium values of respectively the logarithm of the entry and choice thresholds, the marginal costs at the entry and choice thresholds and the average degrees of fragmentation as functions of the probability of successful provision $\mathbb{P}(\mathbb{I}=1)=e^{\frac{-1}{\theta}}$.

Figure 5: Entry and choice thresholds φ^{e*} and φ^{SA*} as functions of institutions $\mathbb{P}(\mathbb{I}=1)$

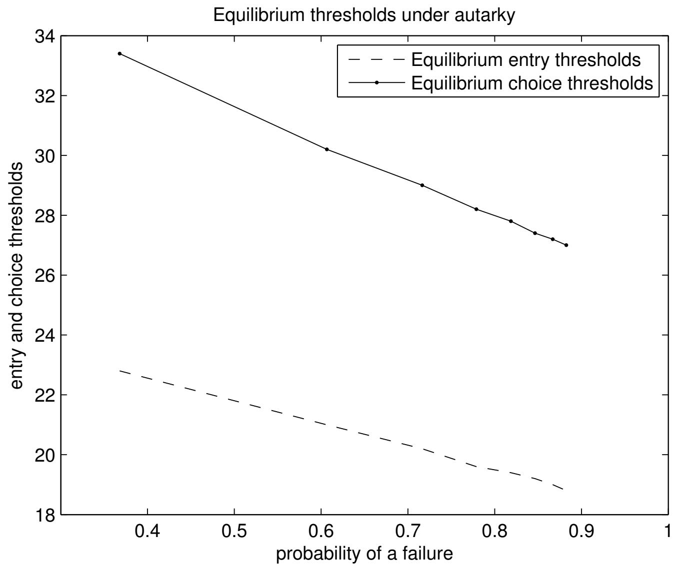
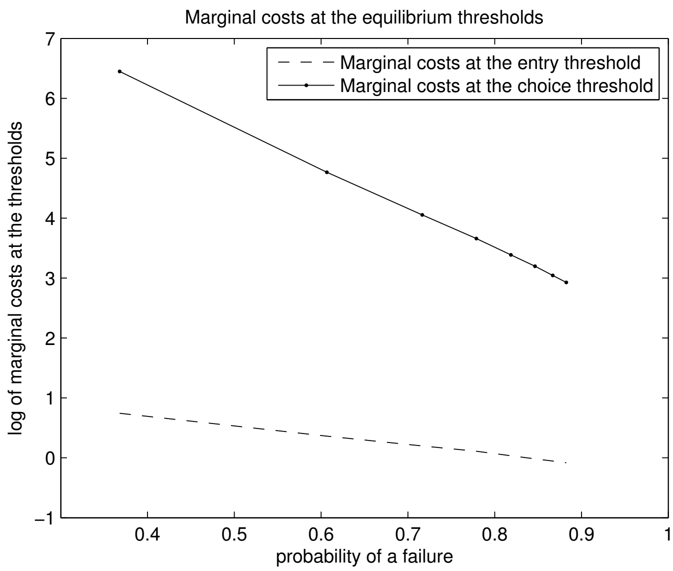
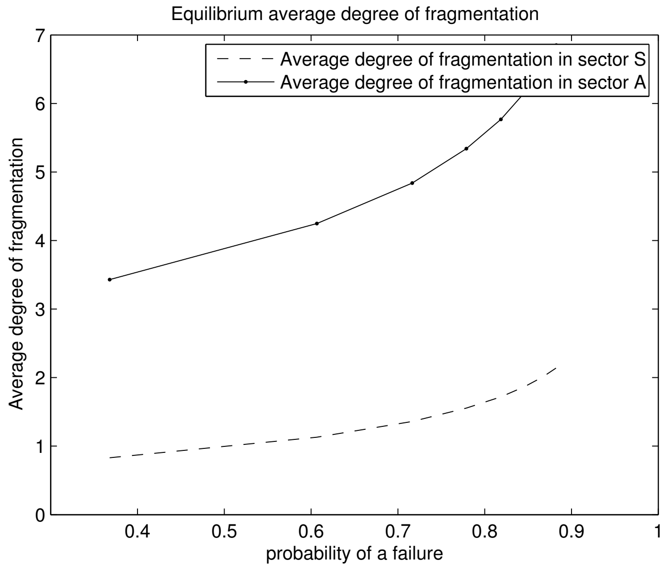


Figure 6: Marginal costs at the productivity thresholds $(\beta^S(\varphi^{e*}),\beta^A(\varphi^{SA*}))$ as functions of institutions $\mathbb{P}(\mathbb{I}=1)$



et al. (2007): final firms' productivity is drawn from a Pareto distribution with scale parameter 1 and shape parameter 3.4; $\sigma=3.8$, $f_e=2$ and $f=0.1$. Moreover we fix the hours endowment $h=1$, number of workers $L=100$, complexity parameters $z^S=10$ and $z^A=40$.

Figure 7: Average degrees of fragmentation $(\tilde{N}^S, \tilde{N}^A)$ as a function of institutions $\mathbb{P}(\mathbb{I} = 1)$



3.3 Equilibrium under free trade

In this section we allow countries to trade varieties of the two goods at no costs. The extension to costly trade has similar results and it is briefly discussed in section 3.4. We assume that countries only differ in their institutional qualities and that country H has better institutions ($\theta^H > \theta^F$). This difference creates a comparative advantage in one of the two sectors. Contrary to a simple Ricardian model with a single firm, the specialization might not be complete even in the case of no trade costs. Finally we assume that workers are not mobile across countries.

In the free trade equilibrium consumers of both countries have access to foreign varieties, i.e. $\forall k \; \forall i$, $\Omega^i_{FT,k} = \Omega^i_k + \Omega^i_{-k}$ where $-k$ is the trade partner country index. The consumers' optimization does not change. Turning to firms, we notice that their optimal organization does not change either. Moreover, the free-trade standard result that all the firms that produce also export holds within our framework as well[22]. We can notice that two final firms with the same productivity level φ in different countries might not have the same behavior, i.e. the same optimal choice of sector and prices. Given the difference in institutional qualities, a firm with the productivity level φ has a marginal cost $\beta^i_H(\varphi)$ in country H and $\beta^i_F(\varphi)$ in country F. Given that country H has better institutions, the marginal cost of a firm with productivity φ is lower in country H for any variety in any of the two sectors.

The outcome of each final firm's production decision is thus a vector of prices, one for the domestic market (d) and the other for the export one (x). As a consequence of constant

[22] This is an implication of consumers' love of variety and the assumption of no trade costs.

elasticity of demand across countries and no trade costs, the two pricing rules will be equal, i.e.

$$p^i_{k,d}(\varphi) = p^i_{k,x}(\varphi) = p^i_k(\varphi) = \frac{\beta^i_k(\varphi)}{\rho} \quad \forall\ k, i$$

Given that all firms export with the same price they charge on the domestic market, we have that the price indexes are equalized across countries:

$$P^i_H = P^i_F \quad \forall i$$

Denoting with $r_{k,d}$ the k firm's revenue from domestic sales, with $r_{k,x}$ the firm's revenue from exports and with R_k the consumers' total revenue, we can write the free trade revenues and profits of a final firm in k with productivity φ active in sector i respectively as

$$r^i_k(\varphi) = r^i_{k,d}(\varphi) + r^i_{k,x}(\varphi) = \frac{R_k}{2}\left[\frac{P^i_k}{p^i_{k,d}(\varphi)}\right]^{\sigma-1} + \frac{R_{-k}}{2}\left[\frac{P^i_{-k}}{p^i_{k,x}(\varphi)}\right]^{\sigma-1} = r^i_{k,d}(\varphi)\left[1 + \frac{R_{-k}}{R_k}\right]$$

$$\pi^i_k(\varphi) = \frac{r^i_k(\varphi)}{\sigma} - f$$

It is immediate to see that Proposition 2 still holds under free trade. Firms' sector-indifference condition defines the choice threshold φ^{SA}_k in both countries. The entry threshold φ^e_k is defined as the productivity level that makes profits in the S sector equal to 0 in country k. The entry and the choice thresholds give the expressions for average marginal costs which are identical to the autarky ones. Notice that the price aggregates are instead different from their autarky counterparts: in fact they take into account the varieties imported from the trading partner and can be written as follows

$$P^i_k = \left\{M^i_k[p^i_k(\tilde{\beta}^i_k)]^{1-\sigma} + M^i_{-k}[p^i_{-k}(\tilde{\beta}^i_{-k})]^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}$$

or

$$P^i_k = (M^i_k)^{\frac{1}{1-\sigma}}\frac{\tilde{\beta}^i_k}{\rho} + (M^i_{-k})^{\frac{1}{1-\sigma}}\frac{\tilde{\beta}^i_{-k}}{\rho}$$

where

$$M^S_k = \frac{[G(\varphi^{SA}_k) - G(\varphi^e_k)]}{[1 - G(\varphi^e_k)]}M_k \quad \text{and} \quad M^A_k = \frac{[1 - G(\varphi^{SA}_k)]}{[1 - G(\varphi^e_k)]}M_k \tag{3.14}$$

Firms' dynamics is clearly unchanged with respect to autarky. Country k steady state stability and the firm entry condition are still

$$[1 - G(\varphi^e_k)]M^e_k = \delta M_k$$

and

$$\begin{aligned}&\frac{f}{\delta}\Bigg\{[G(\varphi_k^{SA})-G(\varphi_k^e)]\Big\{\Big[\frac{\tilde{\beta}_k^S(\varphi_k^e,\varphi_k^{SA})}{\beta_k^S(\varphi_k^e)}\Big]^{1-\sigma}-1\Big\}+\\&+[1-G(\varphi_k^{SA})]\Big\{\Big[\frac{\tilde{\beta}_k^A(\varphi_k^{SA})}{\beta_k^A(\varphi_k^{SA})}\frac{\beta_k^S(\varphi_k^{SA})}{\beta_k^S(\varphi_k^e)}\Big]^{1-\sigma}-1\Big\}\Bigg\}=f_e\end{aligned} \tag{3.15}$$

Goods' market clearing in country k requires that the expenditure share in each i sector equalizes the domestic revenue of k-owned firms producing an i variety plus the revenue made by foreign firms exporting an i variety to k. Mathematically

$$R/2 = R^i_{k,d} + R^i_{-k,x} \quad \forall\ k,i$$

Finally, labor market condition does not change with respect to autarky. We can now state the following

Proposition 4 *(Free trade equilibrium) The free trade equilibrium is defined through the vectors*

$$\{\varphi_k^{e,FT},\varphi_k^{SA,FT},P_k^{S,FT},P_k^{A,FT},M_k^{FT},p_k^{S,FT}(\varphi),p_k^{A,FT}(\varphi)\} \qquad \textit{for } k\in\{H,F\} \tag{3.16}$$

that verify the optimal behaviours of the consumers and the firms, the labor market and good market conditions in each country. The equilibrium under free-trade exists unique.

Proof. See Appendix C.

The first step for the analysis of the free trade equilibrium consists in the derivation of the pattern of comparative advantage which is given in the following

Proposition 5 *(Comparative advantage) Under free trade, the country with better institutions (H) has a comparative advantage in producing varieties in the advanced sector (A).*

Proof. See Appendix C.

3.3.1 Reallocation of resources

A novelty of our paper is the assumption that final firms are mobile across sectors. In fact, not only final firms choose whether to produce, but they also decide which good to produce. The ability of firms to chose their sector introduces a new mechanism through which resources can be reallocated across firms and sectors.

The reallocation towards more productive firms of resources that were used in autarky by the least productive firms that exit in free-trade, what we call "Melitz effect", is the only channel for the reallocation of resources in papers such as Melitz (2003) and Bernard et al. (2007). In Melitz (2003) resources are limited and reallocated towards better firms and so aggregate productivity increases. In Bernard et al. (2007) resources are reallocated within and across industries. In each sector, firms choose whether to produce but do not choose their sector. The "Melitz effect" takes place in both sectors, and is magnified in the sector with the comparative advantage.

What allows us to have different results with respect to Bernard et al. (2007) is the assumption that the free-entry condition is not a condition per sector but a condition for the whole economy.[23] In our model, new export opportunities do not necessarily lead to a higher entry threshold.

The reallocation of resources depends on whether firms exit or enter the production process compared to autarky, which in turns crucially depends on which good the active final firms choose to produce. In general, if the free trade equilibrium entry threshold increases with respect to autarky, resources are reallocated to more productive firms, the so-called "Melitz effect". A decrease in the equilibrium entry threshold instead leads to a decrease in the whole aggregate productivity and this is what we call an "anti Melitz effect".

The sector choice introduces another dimension to the analysis of the effects of trade on productivity, both at the sector and at the aggregate level. The comparative advantage dynamics, through changes in the relative price, drives the choice of sector. If the equilibrium choice threshold decreases, firms that were producing in the simple sector in autarky now produce in the advanced sector and resources are reallocated from the simple to the advanced sector. We start looking at the advanced sector, where the effect of trade on productivity depends solely on the movements of the choice threshold. This effect is described in the following

Proposition 6 *(Aggregate productivity in A) The free trade aggregate productivity in the advanced sector (A) decreases in the country with the comparative advantage in the advanced sector, and increases in the other country compared to autarky.*

Proof. See Appendix C.

We provide a graphical representation of Proposition 6 in Figure 8 and Figure 9.

Figure 8: Change in thresholds for the country with good institutions

[23] The free-entry condition is the expression that drives the results in Melitz (2003) and Bernard et al. (2007). This condition requires the average profit to be equal to the entry cost. The intuition of the result is that higher profit opportunities due to exports lead to a higher entry threshold that reduces the average price in equilibrium.

Figure 9: Change in thresholds for the country with poor institutions

The result from Proposition 6 is driven by the choice of firms to produce in one of the two sectors. This choice depends on the comparative advantage of the country. The country with the good institutions has a comparative advantage in the advanced sector and the relative price of the advanced good increases. Firms that were previously producing in the simple sector decide to produce in the advanced sector and get higher profits, and firms with lower productivity φ thus enters the advanced sector. In the other country, the opposite happens and some firms that were previously producing in the advanced sector decide to produce in the simple sector. Firms with higher productivity φ thus decides to produce in the simple sector.

What are the implications of this result for the productivity in the simple sectors and, most importantly, for the aggregate productivity of the two countries? Due to the complexity of our modelling framework we are not able to derive an analytical answer to this question and we need to rely upon a numerical simulation of the equilibrium. Nevertheless, Proposition 6 reveals a mechanism that will guide our economic intuition.

Consider country H with good intitutions. The pattern of comparative advantage attracts the final firms into the advanced sector and therefore there are firms that would have produced the simple goods under autarky but produce the advanced goods under free trade. Ceteris paribus, higher complexity of the good calls for higher 'consumption' of resources (higher fragmentation of production). Moreover, final firms in this bigger advanced sector benefit from the highest export opportunities, this again calls for higher 'consumption' of resources. Given inter industry reallocation of final producers, the final firms above the free trade entry threshold are consuming more resources than what they would have done under autarky. This mechanisms suggests that the resources available for the firms below the free trade choice threshold could be less than what they would have been under autarky. There are other general equilibrium mechanisms that affect the movement of the entry threshold and that we are not able to capture analytically, but the result in Proposition 6 are consistent with an increase in the entry threshold for country H or, in other words, with a "Melitz effect".

When instead the pattern of comparative advantage attracts firms into the simple sector (in the country with weak institutions), free trade has the opposite effects on resources allocation. On the one hand, all final firms can export and this calls for a higher consumption of resources. On the other hand, the pattern of comparative advantage is such that under free trade there are firms that would have produced an advanced variety under autarky but produce a simple one under free trade. The reduced complexity decreases the degree of fragmentation and, ceteris paribus, the consumption of resources. Those two effects on total resources consumption have opposite sign. In the case of country F, the result in Proposition 6 suggests an ambiguous movement of the entry threshold, or in other words, a possible "anti-Melitz effect".

3.3.2 Numerical analysis of the Free-Trade Equilibrium

Due to the analytical complexity of the model it is not possible to explicitly characterize the key components of the tree-trade Equilibrium. We thus turn to a parametric version of the equilibrium. This exercise has two purposes. First, it allows us to get additional results in terms of aggregate productivity and welfare. Second, it enables us to assess the role of institutional proximity on production, sector choices, and trade. The parametrization of the equilibrium follows the numerical exercise in Bernard et al. (2007), and we check our main results for a large range of complexity and institutional parameters[24]. For the following exercise, we assume that country H has the best institutions ($\theta^H > \theta^F$).

Relative prices

Result 1 *The gap between the autarky relative prices and the free-trade relative price decreases in the institutional proximity.*

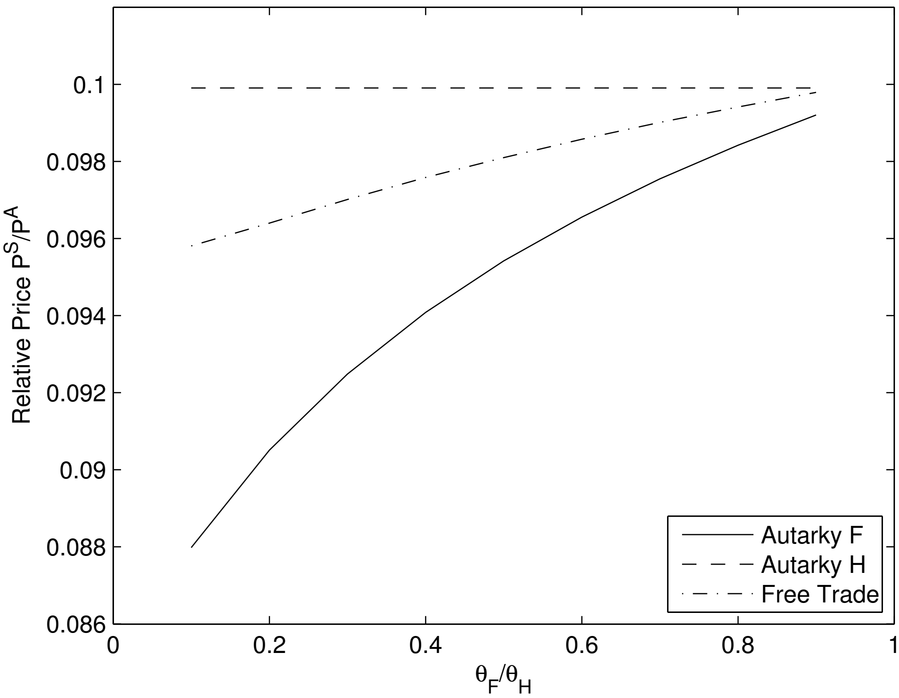
This result is an illustration of the comparative advantage dynamics and its effect on relative price convergence. Figure 10 shows the equilibrium relative price P^S/P^A as a function of the ratio θ_H/θ_F which we interpret as an indicator of *institutional proximity*[25]. Institutional heterogeneity is a source of comparative advantage and the country with the best institutions develops a comparative advantage in the advanced sector. Figure 10 shows that the difference between the autarky relative prices in the two countries decreases with the institutional proximity. The middle line represents the free-trade relative price. For large gaps between the autarky relative price and the free-trade price, more firms change sectors. In country H, the relative price of the advanced good increases so more firms choose to produce the advanced good whereas in country F the relative price of the simple good increases so more firms choose to produce the simple good.

Aggregate productivity

[24] All the details of our parametrization are reported in Appendix D.

[25] Variation in θ_H/θ_F is obtained fixing θ_F and letting θ_H increase. By construction, our measure of institutional proximity is also a function of the parameter θ_F and therefore has to be interpreted as conditional on the fixed value of θ_H that we choose for our numerical exercise.

Figure 10: Relative price P^S/P^A



Proposition 6 only gives results for the aggregate productivity in the advanced sector. Our parametrization delivers numerical results for changes in the two thresholds, the entry and the choice, and for changes in aggregate productivity in the two sectors going from autarky to free-trade. The left diagram of Figure 11 plots on the vertical axis the *entry ratio*, defined as the entry threshold under autarky over the entry threshold under free trade ($\varphi^e(Aut)/\varphi^e(FT)$), for both countries. The right diagram instead shows the *choice ratio*, defined as the ratio between the choice threshold under autarky and the choice threshold under free trade ($\varphi^{SA}(Aut)/\varphi^{SA}(FT)$).

Result 2 *In the country with the best institutions, and the comparative advantage in the advanced sector, the aggregate productivity in the advanced sector (A) decreases but the whole aggregate productivity increases.*

In the country with good institutions, for any level of institutional proximity, the free-trade entry threshold, the level of productivity below which firms in F decide not to produce, increases. This is consistent with the pro-competitive effect of trade liberalization from Melitz (2003) and Bernard et al. (2007). Export opportunities and the reallocation of firms across sectors increase the average profit. Indeed country H has a comparative advantage in sector A, more firms decide to produce in sector A and the aggregate productivity of sector A decreases (Proposition 6). This implies that the aggregate price of sector A increases and the profits of the new firms in this sector as well as the profits of the previous

Figure 11: Entry and choice ratio

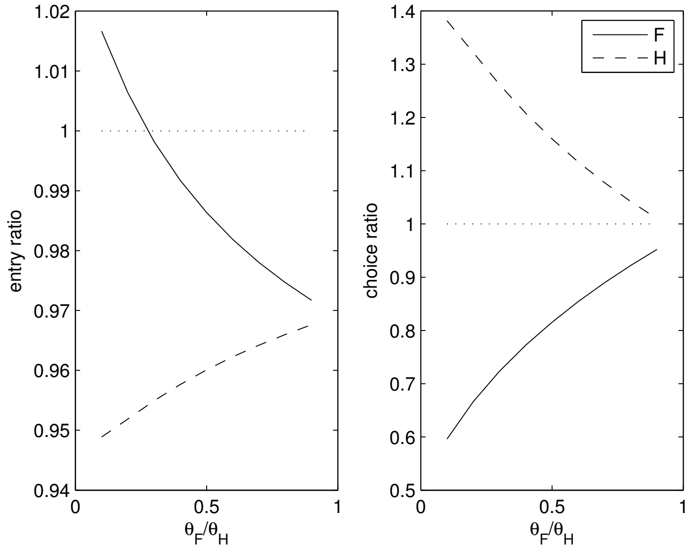
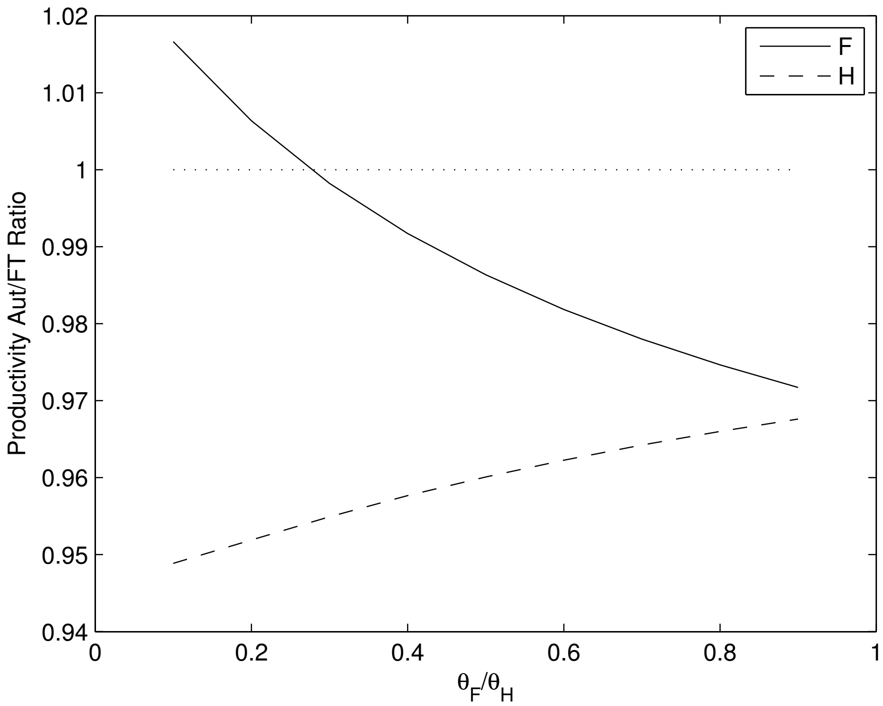


Figure 12: Aggregate productivity (Autarky/Free Trade ratio)



ones increase. Using the free-entry condition, profits of firms in sector S decrease at the equilibrium. In the free trade equilibrium, the least productive firms do not produce any more compared to autarky, and the aggregate price of good S decreases.

Result 3 *In the country with the worst institutions, and the comparative advantage in the simple sector, the aggregate productivity in the advanced sector (A) increases but the whole aggregate productivity decreases (increases) for a low (high) institutional proximity.*

Contrary to country H, there exist institutional parameters for which the entry threshold decreases, what we denoted "the anti-Melitz effect". Figure 11 shows that a low institutional proximity leads to a decrease in the entry threshold. In other words, if the quality of institutions in country F is too low compared to the quality of institutions in country H, free-trade decreases the whole aggregate productivity in country F but increases the whole aggregate productivity in country H compared to autarky. The reasoning is similar to the one for country H. First new export opportunities increase the average profit. Second country F has a comparative advantage in sector S, more firms decide to produce in sector S and the aggregate productivity of sector A increases (Proposition 6). This implies that the aggregate price of sector A decreases and the profits of the firms in this sector decrease. The equilibrium effect on prices in sector S is undetermined and depend on the institutional proximity. When countries are similar the variation of the relative price is lower, and fewer firms change sectors. When countries are very different in terms of institutional quality a lot of firms change sectors, and the average profit in sector A decreases a lot. If the fall is sharp enough, the equilibrium effect is to get increasing profits in sector S. This implies a higher aggregate price in sector S and explains why low-productivity firms start producing. In that case free-trade leads worst firms to start producing and some resources are reallocated from more productive firms towards these new firms.

Welfare of Consumers

In a simple Ricardian framework, trade and the comparative advantage dynamics benefit both countries. Adding heterogeneous firms and reallocations of firms across sectors challenges this result, and creates cases for which welfare, measured here as the real consumption wage, decreases in free-trade compared to autarky.[26]

Result 4 *(i) In the country with the best institutions, and the comparative advantage in the advanced sector, the real wage decreases compared to autarky when the institutional proximity is low. (ii) In the country with the worst institutions, the real wage always increases compared to autarky.*

First the real wage is the same for both countries in free-trade by construction. Then

[26] In the derivation of these results, we do not take into account the love for diversity of consumers.

Figure 13: Real Consumption Wage

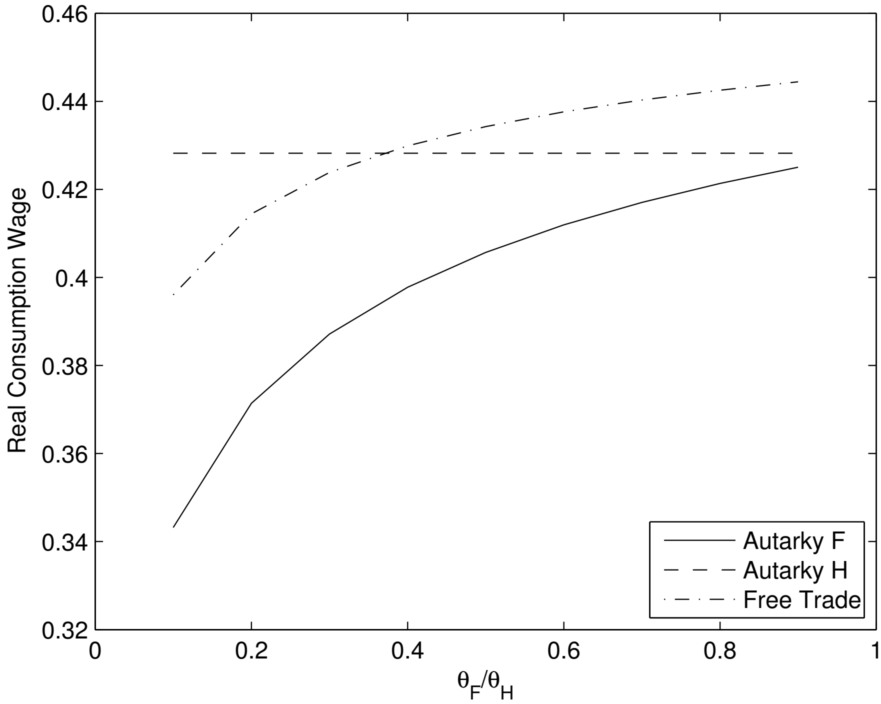
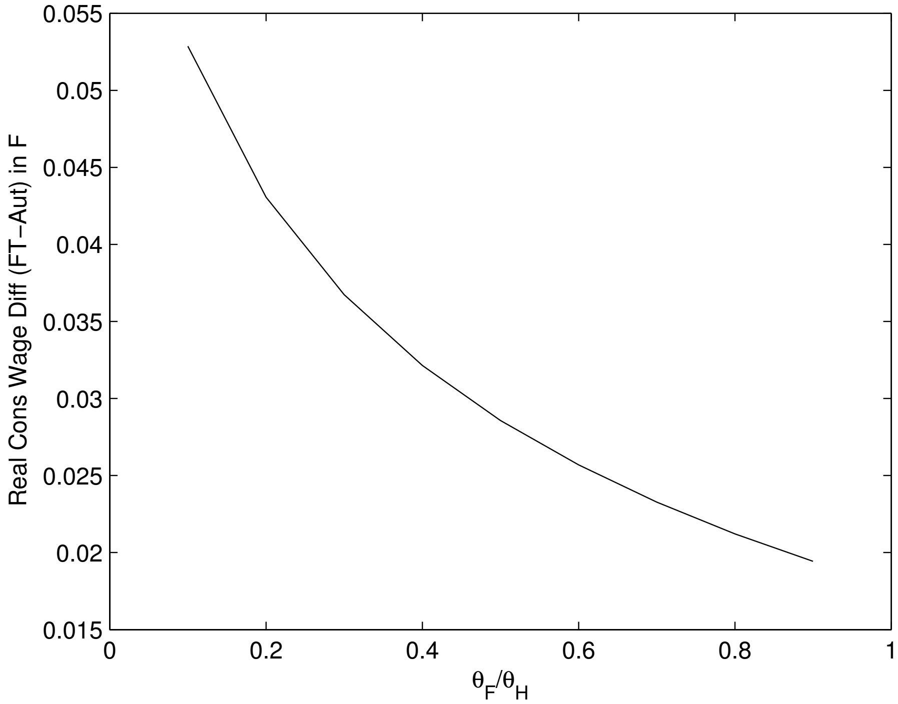


Figure 13 shows that the real wage in the country with the worst institutions (country F) in free-trade is always higher than the real wage in autarky. Consumers in country F benefit from the opening to trade. The fall in aggregate productivity in country F is compensated by access to cheap varieties from country H. On the contrary, the real wage in country H in free-trade is either higher or lower than the real wage in autarky. It is lower for low institutional proximity values. Thus the fall in aggregate productivity in country F directly affects the aggregate price of imports in country H due to the comparative advantage dynamics and the preference for diversity. When the institutional proximity is low, the specialization due to comparative advantage is strong and consumers in country H buy a lot of varieties of good S from country F. Consumers from country H do not always benefit from free-trade in terms of real wage.

Result 5 *In the country with the worst institutions, the welfare gains in terms of real wages are always positive but decrease in the institutional proximity.*

Figure 14 shows that the difference between the free-trade real wage and the autarky real wage decreases in the institutional proximity. When we only focus on real wage, the welfare impact depends more on the comparative advantage dynamics than on the access to more varieties. When the institutional proximity is low, the potential gains from the specialization due to comparative advantage are high (large differences in relative prices) and country F benefits a lot from this specialization.

Figure 14: Free Trade Welfare effect in country H



One limit to this analysis of the real wage is our assumption of a fixed wage due to the standard homogeneous good assumption that freezes the wage channel in the free-trade general equilibrium.

Institutional proximity and industrial composition

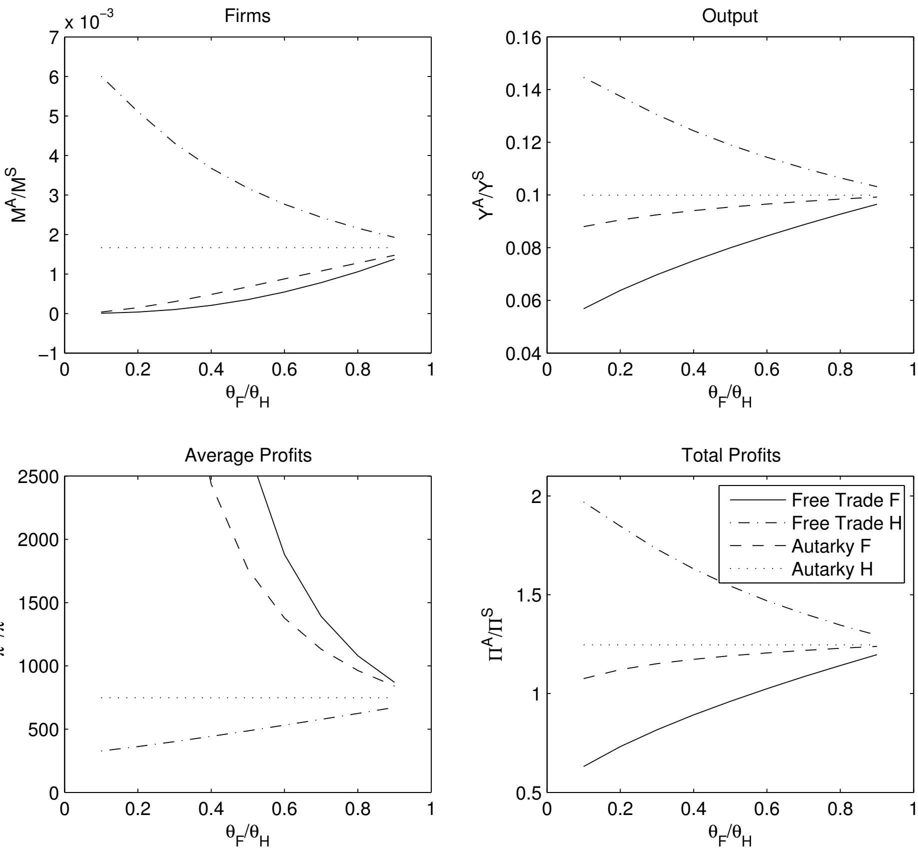
A nice feature of our model with institutional heterogeneity and endogenous production choices is that we can study the impact of institutional convergence on the production structure of both countries in autarky and free-trade. Figure 15 presents the results of this comparative statics exercise.

Result 6 *In the country with the best institutions, (i) the relative mass of firms in the* advanced *sector and the relative production are always higher in free-trade but decrease in the institutional proximity, (ii) the relative average profit in the* advanced *sector is lower in free-trade but the relative total profits are higher.*

Result 7 *In the country with the worst institutions, (i) the relative mass of firms in the* simple sector *and the relative production are always higher in free-trade but decrease in the institutional proximity, (ii) the relative average profit in the* simple *sector is lower in free-trade but the relative total profits are higher.*

All results of this section are symmetric for each country depending on their comparative advantage sector. Figure 15 shows that the sector with the comparative advantage is rel-

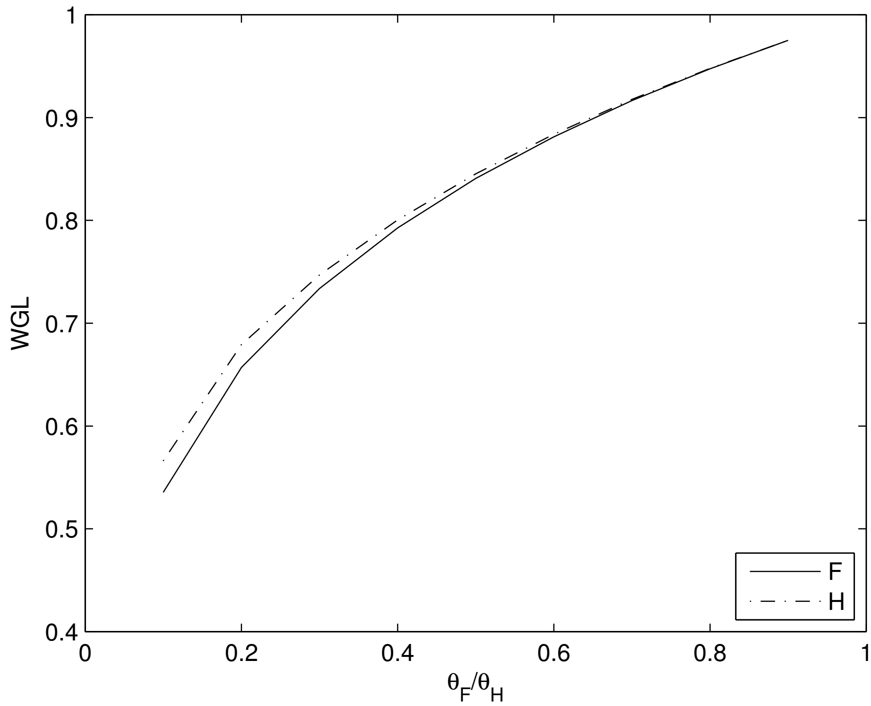
Figure 15: Industrial composition



atively the largest in terms of mass of firms, production and total profits. The differences in the characteristics of sectors are amplified when countries are very different and the gains from specialization potentially high. The results of the average profits follows from Proposition 6 that states that the aggregate productivity decreases in sector A in country H whereas it increases in country F. Thus the relative average profit in sector A increases in free-trade in country F but decreases in country H.

When the countries are similar, trade is not driven by specialization due to their comparative advantage. Consumers' love for diversity is the engine of trade and becomes characterized mainly by intra-industry trade. Figure 16 shows an output-weighted average of the Grubel Lloyd industry indexes, denoted as WGL[27]. Not surprisingly, trade is driven by specialization when differences between countries are high, and increasingly becomes intra-industry the higher the institutional proximity between the two countries.

Figure 16: Intra-industry trade



[27]We computed a weighted version of the Grubel-Lloyd index (see Grubel and Lloyd (1975)) as

$$WGL_k = \sum_{i \in \{S,A\}} \frac{EX_k^i + IM_k^i - |EX_k^i - IM_k^i|}{EX_k^i + IM_k^i} \times \frac{Y_k^i}{Y_k}$$

where weights are the ratio of incomes $\frac{Y_k^i}{Y_k}$.

3.4 Costly trade

All the results and simulations above have been assuming that exporting does not require any additional cost. As an extension, we also derived the main propositions when exporting firms have to pay a variable and a fixed costs to export. The results are very similar to the free trade case with a few caveats.[28]

Compared to the free-trade equilibrium, the presence of fixed costs to export imply that not all the firms export. Therefore, the costly trade equilibrium can be defined similarly to the free trade equilibrium with the addition of two new thresholds that define the productivity thresholds for the exporting firms.

The pattern of comparative advantage under costly trade is also the same as in free trade, i.e. the country with the best (worst) institutions has a comparative advantage in the advanced (simple) sector. However, the specialization is somewhat more extreme: the country with a comparative advantage in the advanced sector only exports in the advanced sector whereas the other country exports in both sectors.

On the other hand, the asymmetric effect of trade on productivity is more nuanced. While the aggregate productivity in the country with the best institutions increases, the effect of trade opening on the aggregate productivity in the country with weak institutions is ambiguous.

4 Conclusions

The empirical trade literature has recently suggested that the benefits of free trade depend on the existence of other non-trade distortions. We provide a theoretical framework in which weak institutions create distortions and hamper the creation of gains from trade in terms of aggregate productivity and welfare.

This is certainly not the first paper that studies the role of institutions in intentional trade. However we introduce some novelties in the theoretical framework that allow to derive original implications regarding the effects of trade in countries with weak institutions.

We propose a monopolistic competition model with heterogeneous firms where comparative advantage are determined by the quality of the business environment. Moreover we allow firms to endogenously choose whether to produce a simple or a complex good, if any.

[28] Since the main results still hold, here we only highlight the differences between free and costly trade. A formal definition of the equilibrium and the complete derivation of the results is available upon request.

We first show that most productive firms always choose to produce the more complex good. This result, together with the pattern of comparative advantage triggered by differences in institutions, determine the reallocation of resource when moving from autarky to free trade which ultimately affect the distribution of the gains from trade.

Our paper confirms a positive effect of trade on the aggregate productivity in the country with good institutions. However the effects of trade in a country lacking in business friendly institutions can be negative. Moreover, the asymmetric effects are amplified when the difference in institutions is very high and trade mainly happens across industries.

The complexity of the model prevents us from deriving all the results analytically, thus we need to rely on numerical simulations. Moreover, we exploit numerical simulations also for the analysis of the industrial composition of the two countries. Finally, the main results are shown to be qualitatively the same in costly trade.

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A Data and methodology

Productivity and trade data

In order to construct measures of productivity, we exploit the data from the World Bank Enterprise Survey. Starting in 2002, the World Bank collects firm level data in its Enterprise Survey dataset. The Enterprise Survey is a firm-level survey of a representative sample of an economy's private sector. The survey covers more than 130 developing and emerging countries in different years between 2002 and 2014. The survey provides detailed information about firms' activity such as sales and other economic variables allowing us to construct a measure of productivity for each firm. Information about the industry in which each firm operates is available at the division level (two digits) of the International Standard Industrial Classification (ISIC Rev. 3).

An additional advantage of the Enterprise Survey is that most of the countries had been surveyed at least twice, therefore we can look at the evolution of aggregate industry productivity across time. In particular, all CIS countries except Tajikistan have been surveyed at least twice by the World Bank. For our purposes we use the 2008 and 2013 surveys for Belarus and Ukraine, 2009 and 2013 for Armenia, Kazakhstan, Kyrgyzstan and Moldova and 2009 and 2012 for Russia. All these surveys a part from Russia in 2012 fall before or after the year of entry into force of the CIS-FTA.

We construct a measure of firms' productivity using the methodology outlined in the paper by Saliola and Seker (2012). Essentially we estimate a firm's total factor productivity (TFP) as the residual of a Cobb-Douglas production function with capital, labor and intermediate goods as factor of production. The regression we run is

$$log(Y) = \beta_1 log(K) + \beta_2 log(L) + \beta_3 log(I) + \delta + \epsilon \tag{A.1}$$

where Y is the output of a firm operating in an industry in a country in a particular year, K represents firm's capital, L is labor used by the firm and I are intermediate goods employed by the firm in the production. The World Bank Enterprise Survey provides firm level information that can be associated to output and these factor of production. In particular, output is measured as firms' sales, capital is the replacement value of machinery, vehicles and equipment, labor is the total compensation of workers including wages, and intermediate goods are measured as the cost of raw and intermediate materials.

In our baseline regression, we run a pooled regression including all available manufacturing firms in all available countries.[29] In order to control for unobservable variables we include a set δ of fixed effects at the country, industry and year level. For each variable in the regression, we exclude the outliers that are more than three standard deviation away from the mean value of the country as in Saliola and Seker (2012).

Using simple OLS we estimate equation A.1 and interpret the residuals ϵ as the TFP of each firm.[30] Productivity at the firm level, is then averaged in order to construct the

[29] The World Bank surveys also services firms. However we restrict our analysis to manufacturing firms in order to match firm level data with trade data.

[30] Given the survey design of the data, we use the sampling weights directly provided by the World Bank. For more information refer to the Methodology page of the Enterprise Survey website:

average productivity of the available industries in each country.[31]

In order to match with firm level data, we retrieve export data at the 2-digits ISIC Rev. 3 from the UN COMTRADE database. For each industry, country and year we construct the revealed comparative advantage (RCA) index (Balassa (1965)) considering only manufacturing goods.[32]

Complexity

In order to classify industries according to complexity, we constructed the PRODY index as defined in Hausmann et al. (2007). The PRODY index gives a sense of the "revealed" technology content of an industry. We calculated the PRODY index using a sample of 133 countries for which we have consistent and reliable trade and GDP data. Trade data is from COMTRADE at the 2 digits ISIC Rev.3 level and GDP per capita is from the World Development Indicators published by the World Bank. Table 2 shows the industries with the largest and smallest values of the index.[33]

Table 2: Smallest and largest PRODY values

	Product Code	ISIC Rev. 3 Product Description	Average PRODY
Smallest	19	Tanning And Dressing Of Leather; Manufacture Of Luggage, Handbags, Saddlery, Harness And Footwear	8637.316
	15	Manufacture Of Food Products And Beverages	9130.748
	16	Manufacture Of Tobacco Products	10410.57
	10	Manufacture Of Wood And Of Products Of Wood And Cork, Except Furniture; etc.	10411.58
	27	Manufacture Of Basic Metals	12063.41
Largest	32	Manufacture Of Radio, Television And Communication Equipment And Apparatus	23177.29
	30	Manufacture Of Office, Accounting And Computing Machinery	23603.89
	29	Manufacture Of Machinery And Equipment N.E.C.	23785.39
	33	Manufacture Of Medical, Precision And Optical Instruments, Watches And Clocks	24530.68
	23	Manufacture Of Coke, Refined Petroleum Products And Nuclear Fuel	25920.47

http://www.enterprisesurveys.org/methodology

[31] In order to calculate the average productivity of the industry we weigh each firm using the share of output of a firm on the total output of the industry in a given year.

[32] This corresponds to industries from 15 to 40 in the ISIC Rev 3.

[33] We averaged the PRODY index in 2006, 2007 and 2008. The full list of 2 digit ISIC industries is available upon request.

B Final firms' organization: a framework for a fully micro-funded application of Costinot' theory

Final firms are indexed with the letter j, suppliers with s and intermediate goods with I. The production of a firm j active in sector i of country k is organized as follows:

- every firm j partitions the sector-specific intermediate goods' space $[0, z^i]$ into N^i_j different product ranges (denote the resulting partition $R^i_j = \{R^i_{k,j}\}_{k=1}^{N^i_j}$), i.e. sets of intermediate goods whose provision is to be assigned to suppliers;
- the firm selects a subset of suppliers, $L^i_j \subset [0, L_k]$. We assume that every supplier can be selected by one firm only. The firm then pays w_k to the supplier irrespectively of the actual provision of intermediate goods;
- for every selected supplier $n \in L^i_j$ and for each unit of the final good $u \in \mathbb{R}^+$, the firm specifies which range R of intermediate goods - if any - has to be provided by that particular supplier for the that particular unit of the final good. Formally the firm designs the mapping

$$O^i_j(\cdot,\cdot) : L^i_j \times \mathbb{R}^+ \rightrightarrows \{R^i_{1,j}, \ldots, R^i_{N^i_j,j}, \varnothing\}$$

From the mapping $O^i_j(\cdot,\cdot)$ we can identify the units of the final-good-variety produced by firm j in sector i for which supplier n provides the intermediate good I. Calling the set of such units $\mathbb{U}^i_j(n, I)$ we have that

$$\mathbb{U}^i_j(n, I) = \{u \in \mathbb{R}^+ \mid \exists\ t \text{ such that } I \in R^i_{t,j} \wedge R^i_{t,j} \in O^i_j(n, u)\}$$

The successful provision indicator is given by

$$S^i_{k,j}(n, I, u) = \begin{cases} 1 & \text{with probability } e^{-\frac{1}{\theta_k}} \\ 0 & \text{with probability } 1 - e^{-\frac{1}{\theta_k}} \end{cases} \tag{B.1}$$

for every $n \in L_k$ and for every pair (u, I) such that $u \in \mathbb{U}^i_j(n, I)$. $S^i_{k,j}(n, I, u) = 1$ means that supplier n is able to provide the intermediate good I for the production of the u^{th} unit of the final good produced by j.

We make the following assumptions:

- a supplier that fails the provision of one intermediate good, fails also in the provision of all the others intermediate goods it was responsible for;
- the firm's organisation applies to all the units of the final good;
- the firms cannot assign more than one supplier to one range of intermediate goods;
- suppliers do not interact among each others.

From this framework we can replicate the following important results that we take as given in the body of the paper.

Result Optimal organization implies that each supplier selected by a final firm provides one and only one range of intermediate goods for every final good?s unit it is responsible for.

Result Each final firm optimally allocates the same number of intermediate goods across ranges.

The proofs of these results consist of the same identical steps of the analogous results in Costinot (2009) and therefore we omit them here.

C Proofs

Proof of Proposition 2 (i) We need to show that the two sector profit functions cross each other once and only once, and that this happens for a positive value of profits. In equilibrium there must be production in both sectors to clear demand. Therefore there must exist two different productivity values φ_1 and φ_2 such that $\pi^S(\varphi_1) > \pi^A(\varphi_1) > 0$ and $\pi^A(\varphi_2) > \pi^S(\varphi_2) > 0$. Given observation 2 we just need to check the sign of the second derivative of the profit functions with respect to productivity. We remove the i index since our computations hold for both industries.

$$\pi'(\varphi) = \frac{R}{2\sigma}(\sigma-1)\Big[\frac{P\rho}{w\beta(\varphi)}\Big]^{\sigma-2} \times \frac{-w\overbrace{\beta'(\varphi)}^{<0}P\rho}{[w\beta(\varphi)]^2} > 0$$

$$\pi''(\varphi) = \frac{R}{2\sigma}(\sigma-1)\Bigg\{(\sigma-2)\Big[\frac{P\rho}{w\beta(\varphi)}\Big]^{\sigma-3}\Big[\frac{-w\beta'(\varphi)P\rho}{[w\beta(\varphi)]^2}\Big]^2 + \overbrace{+\Big[\frac{P\rho}{w\beta(\varphi)}\Big]^{\sigma-2}\Big[\frac{P\rho\{2[\beta'(\varphi)]^2 - \overbrace{\beta''(\varphi)}^{>0}\beta(\varphi)\}}{[w\beta(\varphi)]^3}\Big]}^{>0}\Bigg\} > 0 \quad \text{(C.1)}$$

Given that profit functions are both always convex it must be that if they cross they cross only once.

(ii) Existence in equilibrium of φ^e and φ^{eA} such that $\pi^S(\varphi^e) = \pi^A(\varphi^{eA}) = 0$ is a trivial corollary of Observation 2. We want to prove that $\varphi^e < \varphi^{eA}$. Assume by contradiction that $\varphi^e > \varphi^{eA}$. Then, $\forall\ \varphi^+ > \varphi^{SA}$ we have that $\pi^S(\varphi^+) > \pi^A(\varphi^+)$. Using the profit expression and after some algebra we get the following

$$\pi^S(\varphi^+) > \pi^A(\varphi^+) \iff \frac{P^S}{P^A} > \frac{\beta^S(\varphi^+)}{\beta^A(\varphi^+)} \quad \text{(C.2)}$$

Analogously, $\forall\ \varphi^- < \varphi^{SA}$ we have that $\pi^S(\varphi^-) < \pi^A(\varphi^-)$. As before

$$\pi^S(\varphi^-) < \pi^A(\varphi^-) \iff \frac{P^S}{P^A} < \frac{\beta^S(\varphi^-)}{\beta^A(\varphi^-)} \quad \text{(C.3)}$$

Combining the two conditions (C.2) and (C.3) we get

$$\frac{\beta^S(\varphi^-)}{\beta^A(\varphi^-)} > \frac{\beta^S(\varphi^+)}{\beta^A(\varphi^+)} \quad \text{(C.4)}$$

Defining the function $B(\varphi) := \frac{\beta^S(\varphi)}{\beta^A(\varphi)}$ we can show that $B'(\varphi) > 0$. This contradicts condition (C.4) and completes the proof.

(iii) From (i), (ii) and profit maximisation. ■

Proof of Proposition 3

Detailed derivation of the Autarky equilibrium conditions

Average profits as functions of the entry and choice thresholds The average profits in the two sectors are defined by the following expressions:

$$\bar{\pi}^S = \frac{\int_{\varphi^e}^{\varphi^{SA}} \pi^S(\varphi) g(\varphi) \mathrm{d}\varphi}{[G(\varphi^{SA}) - G(\varphi^e)]}$$

$$\bar{\pi}^A = \frac{\int_{\varphi^{SA}}^{\infty} \pi^A(\varphi) g(\varphi) \mathrm{d}\varphi}{[1 - G(\varphi^{SA})]}$$

We can now derive average profits as functions of the productivity cutoffs:

$$\bar{r}^S = \underbrace{r^S(\tilde{\beta}^S(\varphi^e, \varphi^{SA}))}_{\text{or } r^S(\tilde{\beta}^S)} = \left[\frac{\tilde{\beta}^S(\varphi^e, \varphi^{SA})}{\beta^S(\varphi^e)}\right]^{1-\sigma} \underbrace{r^S(\beta^S(\varphi^e))}_{\text{or } r^S(\varphi^e)} \tag{C.5}$$

$$\bar{r}^A = \underbrace{r^A(\tilde{\beta}^A(\varphi^{SA}))}_{\text{or } r^A(\tilde{\beta}^A)} = \left[\frac{\tilde{\beta}^A(\varphi^{SA})}{\beta^A(\varphi^{SA})}\right]^{1-\sigma} \underbrace{r^A(\beta^A(\varphi^{SA}))}_{\text{or } r^S(\varphi^{SA})}$$

and

$$\bar{\pi}^S = \pi^S(\tilde{\beta}^S) = \left[\frac{\tilde{\beta}^S(\varphi^e, \varphi^{SA})}{\beta^S(\varphi^e)}\right]^{1-\sigma} \frac{r^S(\varphi^e)}{\sigma} - f$$

$$\bar{\pi}^A = \pi^A(\tilde{\beta}^A) = \left[\frac{\tilde{\beta}^A(\varphi^{SA})}{\beta^A(\varphi^{SA})}\right]^{1-\sigma} \frac{r^A(\varphi^{SA})}{\sigma} - f$$

We still need an expression for $r^S(\varphi^e)$ and $r^A(\varphi^{SA})$ to reach our goal. We use the definitions of φ^e and φ^{SA}:

$$\pi^S(\varphi^e) = 0 \iff r^S(\varphi^e) = \sigma f$$

$$\pi^S(\varphi^{SA}) = \pi^A(\varphi^{SA}) \iff r^S(\varphi^{SA}) = r^A(\varphi^{SA})$$

Moreover, we notice that the revenue ratio of any two firms φ and φ' in sector i becomes

$$\frac{r^i(\varphi)}{r^i(\varphi')} = \left(\frac{\beta^i(\varphi')}{\beta^i(\varphi)}\right)^{(\sigma-1)} \tag{C.6}$$

Using the revenue ratio (C.6) we can substitute $r^S(\varphi^{SA})$ with $r^S(\varphi^e)[\beta^S(\varphi^e)/\beta^S(\varphi^{SA})]^{\sigma-1}$. Rearranging and substituting $r^S(\varphi^e) = \sigma f$ we get

$$r^A(\varphi^{SA}) = \Big[\frac{\beta^S(\varphi^{SA})}{\beta^S(\varphi^e)}\Big]^{1-\sigma}\sigma f$$

Eventually we can write average profits as

$$\bar{\pi}^S = f\Big\{\Big[\frac{\tilde{\beta}^S(\varphi^e,\varphi^{SA})}{\beta^S(\varphi^e)}\Big]^{1-\sigma} - 1\Big\} \tag{C.7}$$

$$\bar{\pi}^A = f\left\{\left[\frac{\tilde{\beta}^A(\varphi^{SA})}{\beta^A(\varphi^{SA})}\frac{\beta^S(\varphi^{SA})}{\beta^S(\varphi^e)}\right]^{1-\sigma} - 1\right\}$$

Threshold (φ^{SA}) The choice threshold φ^{SA} is defined as the level of productivity that makes a final firm indifferent across sectors, i.e. such that

$$\pi^S(\varphi^{SA}) = \pi^A(\varphi^{SA})$$

which, using the expression for profits, becomes

$$\Big\{\frac{P^S}{\beta^S(\varphi^{SA})}\Big\}^{\sigma-1} = \Big\{\frac{P^A}{\beta^A(\varphi^{SA})}\Big\}^{\sigma-1}$$

using the aggregate price expressions and substituting the sectoral mass of firms we get

$$\Big\{\frac{\tilde{\beta}^S(\varphi^e,\varphi^{SA})}{\beta^S(\varphi^{SA})}\Big\}^{\sigma-1}\frac{[1-G(\varphi^e)]}{M[G(\varphi^{SA})-G(\varphi^e)]} = \Big\{\frac{\tilde{\beta}^A(\varphi^{SA})}{\beta^A(\varphi^{SA})}\Big\}^{\sigma-1}\frac{[1-G(\varphi^e)]}{M[1-G(\varphi^{SA})]}$$

and rearranging

$$\frac{\tilde{\beta}^S(\varphi^e,\varphi^{SA})^{\sigma-1}}{G(\varphi^{SA})-G(\varphi^e)} = \Big[\frac{\beta^S(\varphi^{SA})}{\beta^A(\varphi^{SA})}\Big]^{\sigma-1}\frac{\tilde{\beta}^A(\varphi^{SA})^{\sigma-1}}{1-G(\varphi^{SA})} \tag{φ^{SA}}$$

The free-entry condition (FE) Given the firms dynamics as described in Melitz (2003) we derive the firm entry condition:

$$V = \frac{[1-G(\varphi^e)]}{\delta}\bar{\pi} = f_e \tag{C.8}$$

with V being the ex-ante (before the productivity realization) utility of the final firm, $\bar{\pi}$ the average ex-post profit in the economy and f_e the fixed cost that has to be paid initially to draw a productivity level. Decomposing the aggregate average profits we can rewrite the LHS of the above equation:

$$\frac{1}{\delta}\Big[[G(\varphi^{SA})-G(\varphi^e)]\bar{\pi}^S + [1-G(\varphi^{SA})]\bar{\pi}^A\Big] = f_e \tag{C.9}$$

Using the expressions for average profits (C.7) and (C) in the two sectors we have:

$$\begin{aligned}\frac{f}{\delta}\Bigg\{&[G(\varphi^{SA})-G(\varphi^e)]\Big\{\Big[\tfrac{\tilde{\beta}^S(\varphi^e,\varphi^{SA})}{\beta^S(\varphi^e)}\Big]^{1-\sigma}-1\Big\}+\\ &+[1-G(\varphi^{SA})]\Big\{\Big[\tfrac{\tilde{\beta}^A(\varphi^{SA})}{\beta^A(\varphi^{SA})}\tfrac{\beta^S(\varphi^{SA})}{\beta^S(\varphi^e)}\Big]^{1-\sigma}-1\Big\}\Bigg\} = f_e\end{aligned} \tag{C.10}$$

We use equation (ϕ^{SA}) to derive an expression for $\left\{\frac{\tilde{\beta}^A(\varphi^{SA})\beta^S(\varphi^{SA})}{\beta^A(\varphi^{SA})}\right\}^{1-\sigma}$, in particular we get

$$\left\{\frac{\tilde{\beta}^A(\varphi^{SA})\beta^S(\varphi^{SA})}{\beta^A(\varphi^{SA})}\right\}^{1-\sigma} = \frac{[G(\varphi^{SA}) - G(\varphi^e)]}{[1 - G(\varphi^{SA})]}\left\{\tilde{\beta}^S(\varphi^e, \varphi^{SA})\right\}^{1-\sigma} \tag{C.11}$$

We get:

$$\frac{f}{\delta}\Bigg\{[G(\varphi^{SA}) - G(\varphi^e)]\Big\{\Big[\frac{\tilde{\beta}^S(\varphi^e,\varphi^{SA})}{\beta^S(\varphi^e)}\Big]^{1-\sigma} - 1\Big\} +$$
$$+[1 - G(\varphi^{SA})]\Big\{\frac{[G(\varphi^{SA})-G(\varphi^e)]}{[1-G(\varphi^{SA})]}\Big[\frac{\tilde{\beta}^S(\varphi^e,\varphi^{SA})}{\beta^S(\varphi^e)}\Big]^{1-\sigma} - 1\Big\}\Bigg\} = f_e$$

$$\iff \begin{array}{l} \frac{f}{\delta}\Bigg\{2[G(\varphi^{SA}) - G(\varphi^e)]\Big[\frac{\tilde{\beta}^S(\varphi^e,\varphi^{SA})}{\beta^S(\varphi^e)}\Big]^{1-\sigma} \\ -[G(\varphi^{SA}) - G(\varphi^e)] - [1 - G(\varphi^{SA})]\Bigg\} = f_e \end{array}$$

$$\iff \frac{G(\varphi^{SA}) - G(\varphi^e)}{\tilde{\beta}^S(\varphi^e, \varphi^{SA})^{\sigma-1}} = \frac{1}{2}\Big[\delta f_e/f + 1 - G(\varphi^e)\Big]\beta^S(\varphi^e)^{1-\sigma} \tag{FE}$$

The labor market condition We first solve the number of workers/suppliers needed at the equilibrium for the sector X. Given the technology in this sector, $S_x = X = \frac{\alpha_x \mathcal{R}}{p_x}$. With p_x normalized to 1 we have

$$S_x = \alpha_x \mathcal{R} = \alpha_x w L$$

Labor is used to enter the production process as well as to produce. The economy has a population of L workers. S^e denotes the total amount of suppliers used in the entry process which is not sector specific and S_i^p denotes the total amount of suppliers used for production in sector i. The labor market clearing conditions are:

$$S^e + S^p = L - S_x = (1 - \alpha_x)L \quad \text{with} \quad S^p = S_S^p + S_A^p$$

Every period, each firm in sector i, with a productivity level φ needs f plus $\beta^i(\varphi)y^i(\varphi)$ suppliers to produce the quantity $y^i(\varphi)$ of goods. Total production-labor demand in sector i would be

$$S_i^p = M^i \bar{S}_i^p \quad \forall i$$

where L_i^p denotes average production-labor demand in sector i whose expression is

$$\bar{S}_S^p = \frac{1}{[G(\varphi^{SA}) - G(\varphi^e)]}\Bigg[\int_{\varphi^e}^{\varphi^{SA}} \beta^s(\varphi)y^s(\varphi)g(\varphi)d\varphi + f\Bigg]$$

$$\bar{S}_A^p = \frac{1}{[1 - G(\varphi^{SA})]}\Bigg[\int_{\varphi^{SA}}^{\infty} \beta^A(\varphi)y^A(\varphi)g(\varphi)d\varphi + f\Bigg]$$

Given the following expressions for supply and number of final firms

$$y^i(\varphi) = \frac{r^i(\varphi)}{p^i(\varphi)} = \frac{R}{2}\Big[\frac{\rho}{\beta^i(\varphi)}\Big]^{\sigma}(P^i)^{\sigma-1}$$

$$M^S = \frac{[G(\varphi^{SA}) - G(\varphi^e)]}{[1 - G(\varphi^e)]}M \quad M^A = \frac{[1 - G(\varphi^{SA})]}{[1 - G(\varphi^e)]}M$$

the final labor market clearing condition is:

$$\Big(\rho\Big)^{\sigma} \tfrac{M\mathcal{R}}{[1-G(\varphi^e)]}\Big[\alpha_S \int_{\varphi^e}^{\varphi^{SA}} \tfrac{(P^S)^{\sigma-1}}{\beta^S(\varphi)^{\sigma}} g(\varphi)d(\varphi) +$$

$$+\alpha_A \int_{\varphi^{SA}}^{\infty} \tfrac{(P^A)^{\sigma-1}}{\beta^A(\varphi)^{\sigma}} g(\varphi)d(\varphi)\Big] + Mf + M_e f_e = (1 - \alpha_x)L$$

L is exogenously given as the total number of workers in the economy.

Body of the proof The equilibrium thresholds solve the following system of equations:

$$\begin{cases} (FE) \qquad V(\varphi^{e*}, \varphi^{SA*}) = f_e \\ (\text{def } \varphi^{e*}) \qquad \pi^S(\varphi^{e*}) = 0 \\ (\text{def } \varphi^{SA*}) \qquad \pi^S(\varphi^{SA*}) = \pi^A(\varphi^{SA*}) \\ \text{labor and good market clearing conditions} \end{cases} \tag{C.12}$$

All the equilibrium endogenous variables can be pinned down from the vector of thresholds $(\varphi^{e*}, \varphi^{SA*})$. In particular, the number of firms entering and exiting production is given by the stationary equilibrium equation and pined down by the labor market condition.

We need to show that the following system has at least one solution $(\varphi^e_*, \varphi^{SA}_*)$

$$\begin{cases} (\varphi^{SA}) \qquad \frac{\tilde{\beta}^S(\varphi^e,\varphi^{SA})^{\sigma-1}}{G(\varphi^{SA})-G(\varphi^e)} = \Big[\frac{\beta^S(\varphi^{SA})}{\beta^A(\varphi^{SA})}\Big]^{\sigma-1} \frac{\tilde{\beta}^A(\varphi^{SA})^{\sigma-1}}{1-G(\varphi^{SA})} \\ (FE) \qquad \frac{G(\varphi^{SA})-G(\varphi^e)}{\tilde{\beta}^S(\varphi^e,\varphi^{SA})^{\sigma-1}} = \frac{1}{2}\Big[\delta f_e/f + 1 - G(\varphi^e)\Big]\beta^S(\varphi^e)^{1-\sigma} \end{cases} \tag{C.13}$$

Define the right hand side (RHS) of (φ^{SA}) as $h : \varphi^{SA} \longrightarrow h(\varphi^{SA})$. Consider the following:

1. $\Big[\frac{\beta^S(\varphi^{SA})}{\beta^A(\varphi^{SA})}\Big]^{\sigma-1}$ is strictly increasing in φ^{SA};
2. $\frac{\tilde{\beta}^A(\varphi^{SA})^{\sigma-1}}{1-G(\varphi^{SA})} = 1/\int_{\varphi^{SA}}^{\infty} \beta^A(\varphi)^{1-\sigma} g(\varphi)\mathrm{d}\varphi$ is strictly increasing in φ^{SA}.

We conclude that $h'(\varphi^{SA}) > 0$.

Define the RHS of (FE) as $m : \varphi^e \longrightarrow m(\varphi^e)$. If a solution of (C.13) exists it has to satisfy the following equation

$$h(\varphi^{SA}) = \frac{1}{m(\varphi^e)} \tag{C.14}$$

Given the strict monotonicity of h we can use (C.14) to write the equilibrium value of φ^{SA} as a function of φ^e:

$$\varphi^{SA} = h^{-1}\left(\frac{1}{m(\varphi^e)}\right) =: H(\varphi^e). \tag{C.15}$$

We will now show that (C.13) admits at least one solution of the kind $(\varphi^e_*, H(\varphi^e_*))$. Consider (FE) and rewrite it as an equation in the only unknown φ^e using (C.15)

$$k(\varphi^e) := \int_{\varphi^e}^{H(\varphi^e)} \beta^S(\varphi)^{1-\sigma} g(\varphi) \mathrm{d}\varphi - m(\varphi^e) = 0 \tag{C.16}$$

The following properties hold:

1. $k(\cdot)$ is continuous on its domain $[0, +\infty)$;
2. $\lim_{\varphi^e \to 0} k(\varphi^e) \geq 0$;
3. $\lim_{\varphi^e \to \infty} k(\varphi^e) = -\infty$.

We conclude that (C.16) has at least one solution applying the intermediate value theorem to $k(\cdot)$. This implies that also (φ^{SA}) admits at least a solution of the kind $(\varphi^e_*, H(\varphi^e_*))$:

$$(\varphi^{SA}) \iff 1 / \int_{\varphi^e}^{H(\varphi^e)} \beta^S(\varphi)^{1-\sigma} g(\varphi) \mathrm{d}\varphi = h(H(\varphi^e))$$

This completes the proof. ■

Proof of Proposition 4

Detailed derivation of the Free-Trade equilibrium conditions for one country

Average profits as functions of the entry and choice thresholds The same as under autarky.

Threshold (φ^{SA}) The choice threshold φ^{SA} is defined as the level of productivity that makes a final firm indifferent across sectors, i.e. such that

$$\pi^S(\varphi^{SA}) = \pi^A(\varphi^{SA})$$

which, using the expression for profits, becomes

$$\left\{ \frac{P^S}{\beta^S(\varphi^{SA})} \right\}^{\sigma-1} = \left\{ \frac{P^A}{\beta^A(\varphi^{SA})} \right\}^{\sigma-1} \tag{$\varphi^{SA,FT}$}$$

The free-entry condition (FE) Given the firms dynamics as described in Melitz (2003) we derive the firm entry condition:

$$V = \frac{[1 - G(\varphi^e)]}{\delta} \bar{\pi} = f_e \tag{C.17}$$

with V being the ex-ante (before the productivity realization) utility of the final firm, $\bar{\pi}$ the average ex-post profit in the economy and f_e the fixed cost that has to be paid initially to draw a productivity level. Decomposing the aggregate average profits we can rewrite the LHS of the above equation:

$$\frac{1}{\delta} \Big[[G(\varphi^{SA}) - G(\varphi^e)] \bar{\pi}^S + [1 - G(\varphi^{SA})] \bar{\pi}^A \Big] = f_e$$

Using the expressions for average profits (C.7) and (C) in the two sectors we have:

$$\begin{aligned}&\tfrac{f}{\delta}\Bigg\{[G(\varphi^{SA})-G(\varphi^{e})]\Big\{\Big[\tfrac{\tilde{\beta}^{S}(\varphi^{e},\varphi^{SA})}{\beta^{S}(\varphi^{e})}\Big]^{1-\sigma}-1\Big\}+\\ &+[1-G(\varphi^{SA})]\Big\{\Big[\tfrac{\tilde{\beta}^{A}(\varphi^{SA})}{\beta^{A}(\varphi^{SA})}\tfrac{\beta^{S}(\varphi^{SA})}{\beta^{S}(\varphi^{e})}\Big]^{1-\sigma}-1\Big\}\Bigg\}=f_e\end{aligned} \tag{FE, FT}$$

The labor market condition We first solve the number of workers needed at the equilibrium for the sector X. Given the technology in this sector, $S_X = X = \frac{\alpha_X \mathcal{R}}{p_X}$. With p_X normalised to 1 we have

$$S_X = \alpha_X \mathcal{R} = \frac{\alpha_X R}{1-\alpha_X}$$

Moreover the amount of workers needed for the pre-production stage is by construction

$$S^e = M_e f_e$$

where M_e will be given by steady state stability.

The labor market clearing conditions is thus:

$$L = S^e + S^p + S_X \quad \text{with} \quad S^p = S^p_S + S^p_A$$

Every period, each firm in sector i, with a productivity level φ needs f plus $\beta^i(\varphi)y^i(\varphi)$ production units to produce the quantity $y^i(\varphi)$ of goods. Total production-labor demand in sector i would be

$$S^p_i = M^i \bar{S}^p_i \quad \forall i$$

where $\bar{L}^p_i$ denotes average production-labor demand in sector i whose expression is

$$\bar{S}^p_S = \frac{1}{[G(\varphi^{SA})-G(\varphi^e)]}\Bigg[\int_{\varphi^e}^{\varphi^{SA}} \beta^s(\varphi)y^s(\varphi)g(\varphi)\mathrm{d}\varphi + f\Bigg]$$

$$\bar{S}^p_A = \frac{1}{[1-G(\varphi^{SA})]}\Bigg[\int_{\varphi^{SA}}^{\infty} \beta^A(\varphi)y^A(\varphi)g(\varphi)\mathrm{d}\varphi + f\Bigg]$$

Given the following expressions for supply and number of firms

$$y^i(\varphi) = \frac{r^i(\varphi)}{p^i(\varphi)} = \frac{R}{2}\Big[1+\frac{R_{-k}}{R}\Big]\Big[\frac{\rho}{w}\Big]^{\sigma}(P^i)^{\sigma-1}[\beta^i(\varphi)]^{-\sigma}$$

$$M^S = \frac{[G(\varphi^{SA})-G(\varphi^e)]}{[1-G(\varphi^e)]}M \quad M^A = \frac{[1-G(\varphi^{SA})]}{[1-G(\varphi^e)]}M$$

we can write

$$\begin{aligned}S^p_S &= \frac{M}{[1-G(\varphi^e)]}\Bigg[\int_{\varphi^e}^{\varphi^{SA}} \beta^s(\varphi)\frac{R}{2}\Big[1+\frac{R_{-k}}{R}\Big]\Big[\frac{\rho}{w}\Big]^{\sigma}(P^S)^{\sigma-1}[\beta^S(\varphi)]^{-\sigma}g(\varphi)\mathrm{d}\varphi + f\Bigg]\\ &= \frac{M}{[1-G(\varphi^e)]}\Bigg[\frac{R}{2}\Big[1+\frac{R_{-k}}{R}\Big]\Big[\frac{\rho}{w}\Big]^{\sigma}(P^S)^{\sigma-1}\int_{\varphi^e}^{\varphi^{SA}}[\beta^s(\varphi)]^{1-\sigma}g(\varphi)\mathrm{d}\varphi + f\Bigg]\end{aligned}$$

$$= \frac{M}{[1-G(\varphi^e)]}\left[\frac{R}{2}\Big[1+\frac{R_{-k}}{R}\Big]\Big[\frac{\rho}{w}\Big]^{\sigma}(P^S)^{\sigma-1}[\tilde{\beta}^S(\varphi^e,\varphi^{SA})]^{1-\sigma}[G(\varphi^{SA})-G(\varphi^e)]+f\right]$$

$$= \frac{Mf}{[1-G(\varphi^e)]}+\frac{M}{[1-G(\varphi^e)]}\frac{R}{2}\Big[1+\frac{R_{-k}}{R}\Big]\Big[\frac{\rho}{w}\Big]^{\sigma}(P^S)^{\sigma-1}[\tilde{\beta}^S(\varphi^e,\varphi^{SA})]^{1-\sigma}[G(\varphi^{SA})-G(\varphi^e)]$$

Analogously

$$S_A^p = \frac{Mf}{[1-G(\varphi^e)]}+\frac{M}{[1-G(\varphi^e)]}\frac{R}{2}\Big[1+\frac{R_{-k}}{R}\Big]\Big[\frac{\rho}{w}\Big]^{\sigma}(P^A)^{\sigma-1}[\tilde{\beta}^A(\varphi^{SA})]^{1-\sigma}[1-G(\varphi^{SA})]$$

Thus

$$S_S^p + S_A^p = \frac{2Mf}{[1-G(\varphi^e)]}+\frac{M}{[1-G(\varphi^e)]}\frac{R}{2}\Big[1+\frac{R_{-k}}{R}\Big]\Big[\frac{\rho}{w}\Big]^{\sigma}\times$$
$$\times\Big\{\Big[\frac{P^S}{\tilde{\beta}^S(\varphi^e,\varphi^{SA})}\Big]^{\sigma-1}[G(\varphi^{SA})-G(\varphi^e)]+\Big[\frac{P^A}{\tilde{\beta}^A(\varphi^{SA})}\Big]^{\sigma-1}[1-G(\varphi^{SA})]\Big\}$$

Moreover in equilibrium

$$R = w_k(L-S_X)$$

which plugging the expression for L_X and rearranging becomes

$$R = \Big(\frac{1-\alpha_X}{1-\alpha_X+w_k\alpha_X}\Big)w_k L$$

Given our assumptions on the parameters we have that R is the same in both countries. We can thus simplify our production-labor demand expressions

$$S_S^p + S_A^p = \frac{2Mf}{[1-G(\varphi^e)]}+\frac{M}{[1-G(\varphi^e)]}\Big(\frac{1-\alpha_X}{1-\alpha_X+w\alpha_X}\Big)wL\Big[\frac{\rho}{w}\Big]^{\sigma}\times$$
$$\times\Big\{\Big[\frac{P^S}{\tilde{\beta}^S(\varphi^e,\varphi^{SA})}\Big]^{\sigma-1}[G(\varphi^{SA})-G(\varphi^e)]+\Big[\frac{P^A}{\tilde{\beta}^A(\varphi^{SA})}\Big]^{\sigma-1}[1-G(\varphi^{SA})]\Big\}$$

Using the fact that in equilibrium $w=1$ we have

$$S_S^p + S_A^p = \frac{2Mf}{[1-G(\varphi^e)]}+\frac{M(1-\alpha_X)L}{[1-G(\varphi^e)]}(\rho)^{\sigma}\times$$
$$\times\Big\{\Big[\frac{P^S}{\tilde{\beta}^S(\varphi^e,\varphi^{SA})}\Big]^{\sigma-1}[G(\varphi^{SA})-G(\varphi^e)]+\Big[\frac{P^A}{\tilde{\beta}^A(\varphi^{SA})}\Big]^{\sigma-1}[1-G(\varphi^{SA})]\Big\}$$

The final labor market clearing condition for country k is:

$$L - \alpha_X L - M_e f_e = \tfrac{2Mf}{[1-G(\varphi^e)]} + \tfrac{M(1-\alpha_X)L}{[1-G(\varphi^e)]}(\rho)^\sigma \times$$
$$\times \Big\{ \Big[\tfrac{P^S}{\tilde{\beta}^S(\varphi^e,\varphi^{SA})}\Big]^{\sigma-1}[G(\varphi^{SA}) - G(\varphi^e)] + \Big[\tfrac{P^A}{\tilde{\beta}^A(\varphi^{SA})}\Big]^{\sigma-1}[1 - G(\varphi^{SA})]\Big\} \tag{LMC}$$

This equation contains the following unknowns: M, M_e, φ^e, φ^{SA} and the two price aggregates. We can easily replace M_e with M using the steady state stability condition.

Body of the proof Given the above derivations, all the equilibrium quantities can be derived from a system of 8 equations in the following 8 unknowns $\{\varphi^e_H, \varphi^e_F, \varphi^{SA}_H, \varphi^{SA}_F, P^S, P^A, M_H, M_F\}$. The 8 equations are given by $(\phi^{SA,FT})$, (FE, FT) and (LMC) for both countries plus the expression aggregate price indexes for both sectors (they are equal across countries). The system admits one and only one solution. ■

Proof of Proposition 5 We assume that country H has the best institutions. By definition of the choice threshold $\varphi^{SA,k}$ in country k $\in \{H, F\}$, we have:

$$\pi^S_k(\varphi^{SA}_k) = \pi^A_k(\varphi^{SA}_k) \Rightarrow \frac{P^S_k}{P^A_k} = \frac{\beta^S_k(\varphi^{SA}_k)}{\beta^A_k(\varphi^{SA}_k)}$$

The marginal cost ratio $(\beta^S(\varphi)/\beta^A(\varphi))$ is increasing in φ and in θ as shown in the following steps:

$$\frac{\partial(\beta^S/\beta^A)}{\partial\varphi} = \frac{\frac{\partial\beta^S}{\partial\varphi}\beta^A - \frac{\partial\beta^A}{\partial\varphi}\beta^S}{(\beta^A)^2}$$

$$\frac{\partial(\beta^S/\beta^A)}{\partial\varphi} > 0 \iff \frac{\partial\beta^S}{\partial\varphi}\beta^A - \frac{\partial\beta^A}{\partial\varphi}\beta^S > 0 \iff \frac{\partial\beta^S}{\partial\varphi}/\beta^S > \frac{\partial\beta^A}{\partial\varphi}/\beta^A$$

and by the chain rule, given that β^i takes only real, strictly positive values

$$\iff \frac{\partial\ln\beta^S}{\partial\varphi} > \frac{\partial\ln\beta^A}{\partial\varphi} \tag{C.18}$$

Given that a strictly increasing transformation does not change the behaviour of the derivative's sign we have that $\frac{\partial\beta^i}{\partial\varphi} < 0$ implies $\frac{\partial ln\beta^i}{\partial\varphi} < 0$. Moreover,

$$\frac{\partial\ln\beta^i}{\partial\varphi\partial z^i} = \frac{-2\varphi h\theta - z^i - z^i\sqrt{1+\frac{4\varphi h\theta}{z^i}}}{2\varphi^2 h\theta z^i\sqrt{1+\frac{4\varphi h\theta}{z^i}}} < 0$$

We conclude that inequality (C.18) is verified. Analogously we can show that (β^S/β^A) is increasing in θ, given that

$$\frac{\partial \ln \beta^i}{\partial \theta \partial z^i} = \frac{-2\varphi h\theta - z^i + z^i\sqrt{1 + \frac{4\varphi h\theta}{z^i}}}{2\varphi h\theta^2 z^i\sqrt{1 + \frac{4\varphi h\theta}{z^i}}} < 0$$

Given this intermediate result on the marginal cost ratio we have the following inequality under the autarky equilibrium

$$\frac{\beta_S^H(\varphi_H^{SA*})}{\beta_A^H(\varphi_H^{SA*})} > \frac{\beta_S^F(\varphi_F^{SA*})}{\beta_A^F(\varphi_F^{SA*})}$$

Consequently we get $\frac{P_H^S}{P_H^A} > \frac{P_F^S}{P_F^A}$ for the autarky equilibrium. This defines a comparative advantage for country H to produce varieties of the advanced sector (A) and therefore completes the proof. ■

Proof of Proposition 6 Compared to the autarky choice thresholds φ^{SA*}, we can show that the free-trade choice threshold $\varphi^{SA,FT}$ decreases in the country with the comparative advantage in the advanced sector and increases in the other country. We keep assuming that country H has the best institutions and therefore the comparative advantage in sector A. Proposition 5 gives us the following condition

$$\frac{P_F^{S*}}{P_F^{A*}} < \frac{P^{S,FT}}{P^{A,FT}} < \frac{P_H^{S*}}{P_H^{A*}}$$

We use the equality of profits at the choice thresholds in autarky φ^{SA*} and in free-trade $\varphi^{SA,FT}$ for each country

$$\pi_k^S(\varphi_k^{SA*}) = \pi_k^A(\varphi_k^{SA*}) \quad \Rightarrow \quad \frac{P_k^{S*}}{P_k^{A*}} = \frac{\beta_k^S(\varphi_k^{SA*})}{\beta_k^A(\varphi_k^{SA*})}$$

$$\pi_k^S(\varphi_k^{SA,FT}) = \pi_k^A(\varphi_k^{SA,FT}) \quad \Rightarrow \quad \frac{P^{S,FT}}{P^{A,FT}} = \frac{\beta_k^S(\varphi_k^{SA,FT})}{\beta_k^A(\varphi_k^{SA,FT})}$$

and the result that the function β^S/β^A is strictly increasing to get the following implications

$$\frac{P^{S,FT}}{P^{A,FT}} < \frac{P_H^{S*}}{P_H^{A*}} \quad \Rightarrow \quad \frac{\beta_H^S(\varphi_H^{SA,FT})}{\beta_H^A(\varphi_H^{SA,FT})} < \frac{\beta_H^S(\varphi_H^{SA*})}{\beta_H^A(\varphi_H^{SA*})} \quad \Rightarrow \quad \varphi_H^{SA,FT} < \varphi_H^{SA*}$$

The choice threshold is proved to decrease in the country with the comparative advantage in the advanced sector. We use a similar reasoning for the other country. ■

D Technical details for the numerical exercise about the free-trade equilibrium

Given the many similarities of our modelling framework to that in Bernard et al. (2007), our choice of parameters follows closely the numerical exercise in that paper. We assume a Pareto distribution for ex-ante productivity with shape parameter equal to 3.4 and scale parameter equal to 1. We set elasticity of substitution $\sigma = 3.8$, sunk entry costs $f_e = 2$, fixed production cost $f = 0.1$ and probability of exogenous firm death $\delta = 0.025$. Moreover, we posit equal consumers' expenditure share across sectors, which, given the presence in our model of a technical homogeneous good sector, implies $\alpha = 1/3$. We assume the working hours endowment $h = 1$ and the total number of suppliers/workers $L = 100$. In terms of sector complexity we choose $z^A = 40$ and $z^S = 5$. Our results are robust across other levels of complexity proximity across sectors. Finally, we set the level of institutions in the less fragile country F, $\theta_H = 100$. We perform our simulation across values of the θ_F in the closed interval $[10, 90]$. Our results are robust across other levels of institutions, for instance $\theta_H = 10$ and θ_F varying in the interval $[1, 9]$.