The Political Economy of Migrants Integration*

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Abstract

This paper addresses the issue of migrants' integration into a host country economy. We define integration as a set of policies that enhance the opportunities of a disadvantaged migrant community to participate in the host country labor market. We build a simple game theoretic model describing a static economy with no migration decision but with labor mobility across sectors. We derive testable implications on the relationship between the level of integration implemented in a classical Downsian political economy and some key parameters of both the labor market and the demographic composition in the host country.

Keywords: integration; migration; political economy
JEL Classification: J08; J15; J61

1 Introduction

The integration of migrant communities is a primary concern for policy makers in contemporary Europe. Integration policies are fundamental components of any political agenda and they often become one of the mostly publicly debated issues. Many different approaches to integration have been proposed and implemented across European countries.

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Alongside the the analysis of the cultural aspects of integration, it is important to understand how the key economic trade-offs embedded in a migration economy affect the process of integration. The present study represents a first attempt in this direction, with a specific focus on labor market integration.

The few available data shows that there exists significant cross country heterogeneity in the policies affecting migrants' participation in the host country labor market. Figure 1 plots the MIPEX Labor Market Mobility indicator for the year 2014 in the 38 host countries covered by the MIPEX project.[1] This measure reflects the level of integration embedded in policies targeting migrants' access to labor market; access to general support such as public employment offices but also higher education and vocational training; migrants specific support, such as - for instance - targeted work-related trainings or specific bridging/work placement programmes; and workers' rights (in many countries - including the US and the UK - migrant workers are excluded from parts of the social security system). The indicator goes from a value of zero to 100, representing full integration.

In this paper we start our analysis addressing the following question: how does the level of integration affect the utility of heterogeneous native agents? Then, we study how the integration policies that ameliorate the economic conditions of the migrant community are determined in a classical Downsian political economy where only natives vote and they do so maximizing their individual utilities. This allows us to derive testable implications on the relationship between the level of integration implemented and the key parameters of both the labor market and the demographic composition in the host country.

We derive our results from a game theoretic model framed on a static economy with no migration decision. This is a key assumption: the scope of the paper is not to study why agents decide to migrate. Nor to understand how the number and the characteristics of those migrants allowed to enter in the host country are determined. Instead, we will be looking at mature migration economies parametrized by a fixed stock of migrants. The focus of the paper is on the set of policies that ameliorate the economic conditions of the

[1] MIPEX is a joint survey project of the Barcelona Centre for International Affairs and the Migration Policy Group. A complete description of the database as well as the full data in downloadable format can be found at http://www.mipex.eu/.

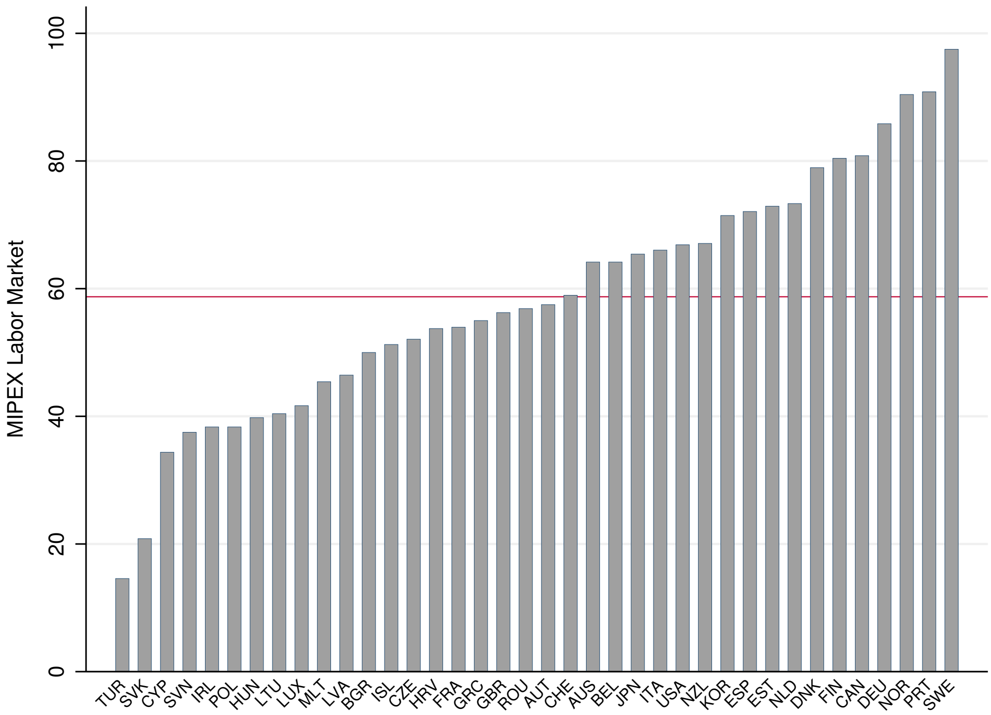

Figure 1: Migrants' integration across countries: labor market focus

disadvantaged migrant minority. In particular, we consider policies affecting labor market access. Non homogeneous opportunities across immigrants and natives workers is an important feature of the labor market in mature host economies: high educated migrants face more difficulties in getting a job than their native-born peers (see OECD/EU, 2015). Integration is represented in the model by more symmetric labor opportunities across native and migrant workers.

In the model the two groups are characterized by a fixed distribution of ability. All the agents compete for vacancies in a two sectors economy. The individual utility of the agents depends upon their ability through the wage and the probability of being hired in the chosen sector. The two sectors differ in terms of the productivity of labor: we call H the high productivity sector and L the low productivity one. In particular, the individual wage that is paid in sector H depends positively upon both the ability of the worker and the average ability of the agents hired in the sector. Moreover, the labor market is such that the probability of being hired in H is a non decreasing function of the individual ability and depends upon an exogenous mass of vacancies K. Instead, any agent competing for a job in sector L is hired with probability one. Finally, migrant workers have to pay a cost in order to compete for a job position in sector H. This cost is a negative function of the level of integration.

The trade-off embedded in our economic environment is simple: higher integration allows the best migrants to compete for better jobs (positions in sector H). On the one hand, this might have positive externalities for some natives due to the wage technology in H. On the other, more able migrants hired in the sector might reduce the probability for some natives to get a position in H. The system of preferences over the policy space (all the possible values of integration) of a native voter incorporates the optimal solution of this trade-off given her individual ability and the strategies of all the other agents in the economy. We will determine the outcome of a classical Downsian political economy aggregating the preferences through simple majority voting.

The model delivers testable implications on the relationship between the level of integration implemented in equilibrium and the parameters of the economy. We find that the level of integration is increasing in the wage scheme offered in sector L. The understanding of this result closely depends upon our interpretation of the L sector. For instance, if we think about L as the informal economy, our theory suggests that when the opportunities in the informal sector are higher the level of integration needed to attract a good migrant worker in sector H, ceteris paribus, should be higher.

Furthermore, we characterize economic environments where the number of vacancies K is a determinant of the integration policy implemented in equilibrium. For any demographic composition of the agents' population, if the number of vacancies is sufficiently low, the maximization of the positive externatilities of migrants' participation requires to leave the less skilled natives out of sector H and to hire more skilled migrants. If this is the case, more vacancies extend the potential positive externalities of migrants participation resulting in higher integration in equilibrium.

Finally, when the number of vacancies is high enough, the positive externalities of migrant's participation are maximised without putting at risk any native job in H. In this case the equilibrium level of integration is an increasing function of the number of migrants.

The rest of the paper is organised as follows. Section 2 discusses related literature; section 3 introduces the model and derives the results; section 4 offers conclusions.

2 Related literature

The paper contributes to the literature on the political economy of migration policies. From the seminal work of Benhabib (1996) many authors have investigated the formation of migration policy platforms. In particular they have been focused on policies concerning the size, the skill and the capital holding composition of migration flows. Ortega (2005) presents a dynamic extension of the model in Benhabib (1996) with heterogeneous skills and voting migrants. In his paper, the mechanics shaping the equilibrium size and skill composition of the migrant community is the trade-off between skill complementarity migration and the shift in political power due to the assumption of voting migrants. Similar results are derived by Dolmas and Huffman (2003). Facchini and Mayda (2008) study an analogous problem using a different approach: the authors consider an international factor mobility model with skilled and unskilled labor. They derive equilibrium quotas and skill composition policies using both a median voter framework and a simple lobbying model. The present paper studies a different set of policy variables, i.e. integration policies. To the best of our knowledge this is the first attempt to address migrants' integration from a political economy perspective. Another key difference in our work is the assumption that, when the migrants communities are in the host country, they do not have the same economic opportunities of the natives. This fundamental feature is missing from previous works on the political economy of migration.

Furthermore, our analysis relates to the theoretical literature studying the processes of integration and segregation. Eguia (2011) builds a model of where the natives set the cost that has to be sustained by the migrants in order to assimilate the social norms required for integration. This cost is chosen strategically as a screening mechanism to select those agents that, within the worse-off group, are willing to assimilate. In our model integration is the direct policy parameter that affects the cost that migrants have to pay in order to compete in the high productivity sector of the economy. Our analysis highlights the side of integration that depends upon economic policies (labor market access) while in Eguia (2011) the focus is on the cultural side of integration, described by the author as the process of adaptation to the norms, the values and the codes of the native population. Another theoretical angle on integration is given in Reich (2011). Her model extends the network formation analysis in Goyal and Vega-Redondo (2005) and studies the intercultural social interaction when both cultural and non cultural activities are available. Her analysis provides a formal understanding of multiculturalism and shades light on the welfare properties of this mode of interaction. While this literature highlights the trade-offs coming from the cultural dimension of the interaction among heterogeneous agents, the present study addresses a purely economic aspect of the issue.

3 The model

3.1 Equilibrium in the labor market

Players and Types There are two populations of agents (workers) in the economy. Denote with $P = \{N, M\}$ the set of populations and call p a generic element of P. N is the population of natives and M of migrants. Each population p is characterized by a continuous mass μ^p. Let us consider strictly positive masses ($\mu^p > 0 \ \forall \ p \in P$), normalize the total mass of agents to 1 ($\sum_{p \in P} \mu^p = 1$) and assume $\mu^N > \mu^M$. Agents are heterogeneous in terms of ability θ. Ability is distributed independently in each population according to a continuous uniform distribution on the support $[0, 1]$. Denote with $F(\cdot)$ the *cdf* of θ and with $f(\cdot)$ its *pdf*.[2]

[2]Looking at the British labor market, Dustmann et al. (2005) have found a remarkable similarity in the skill distributions of the native and migrant workforce. This is supportive of our working assumption of identical ability distributions.

Space of Available Actions Actions represent the sector in which a worker competes for a job. $A_{p,\theta} = \{H, L\} \ \forall \ p \in P$ and $\theta \in [0,1]$. Abilities are common knowledge and the workers simoultaneously decide in which sector to compete for a job. Agents are standard utility maximizers that take into account the labor market structure. In particular, there exists an exogenous hiring technology such that,

- jobs are not scarse in sector L: everyone competing for a job in L is hired;
- there is a continuous mass K of jobs available in sector H. Let us consider the case $\mu_N < K < 1$. The available jobs are allocated among agents competing in H from the most skilled worker downward. Denote the probability of being hired in sector H by p^H. Given perfect information and no frictions in the labor market the probability p^H will be perfectly foreseen by the agents and it will take either value 0 or 1.

Moreover, there exists an exogenous wage technology[3] that pays a striclty positive fixed wage w^L for a job in sector L and w^H for a position in H. We assume that w^H is a function of both the individual ability of the agent and the average ability of the workers employed in H. We denote the average ability by θ^H. Formally,

$$w^H = w\big(\theta, \theta^H\big) \tag{3.1}$$

We assume that $w(\cdot,\cdot)$ is continuous, differentiable and increasing in both arguments.

Strategies Within a given population p, individual strategies are mappings from the type space to the action space. The strategy of an individual i in population p will be

$$s_i^p : [0,1] \longrightarrow \{H, L\} \quad \forall i \text{ and } p \tag{3.2}$$

Let us denote the continuous strategy profile of the agents in population p as S^p. We call instead S^p_{-i} the strategy profile of all the population p agents but i.

[3]Behind this reduced form approach the reader can think of the wage technology as the outcome of a collective bargaining process.

Payoffs For any native worker i of ability θ, the payoffs are given by the following equations

$$U_i^N\left(H, S_{-i}^N, S^M \middle| \theta\right) = w\Big(\theta, \theta^H\left(S^N, S^M\right)\Big) p^H\left(\theta \middle| S^N, S^M\right) \tag{3.3}$$

$$U_i^N\left(L, S_{-i}^N, S^M \middle| \theta\right) = w^L \tag{3.4}$$

For any migrant worker i of ability θ we have instead

$$U_i^M\left(H, S_{-i}^M, S^N \middle| \theta\right) = \Big[w\Big(\theta, \theta^H\left(S^N, S^M\right)\Big) - c(I)\Big] p^H\left(\theta \middle| S^N, S^M\right) \tag{3.5}$$

$$U_i^M\left(L, S_{-i}^M, S^N \middle| \theta\right) = w^L \tag{3.6}$$

The payoffs specification is crucial: it incorporates the disadvantage of migrant workers in competing for a job in H: the migrants have to pay a non negative cost $c(\cdot)$ which is a function of the level of integration I. We take $I \in [0,1]$ where 0 and 1 represent the minimum and maximum integration respectively; we assume $c(\cdot)$ strictly decreasing, with $c(1) = 0$.

In this section we characterise the equilibria in the labor market considering I as an exogenous parameter. In order to do that we need to derive an expression for the functions $p^H\left(\cdot \middle| \cdot, \cdot\right)$ and $\theta^H\left(\cdot, \cdot\right)$. First, denote with $\Theta_H^{s^p} \coloneqq (s^p)^{-1}(H)$ the set of types such that a worker in population p of type $\theta \in \Theta_H^{s^p}$ will be prescribed by strategy s^p to play action H. Assuming Lebesgue-measurability of $\Theta_H^{s^p}$ for any possible strategy s^p we can write the mass of workers in population p that will play H according to s^p as

$$\phi^{s^p} = \mu^p \int_0^1 \mathbb{1}_{\{\Theta_H^{s^p}\}} f(\theta) \mathrm{d}\theta$$

Given the hiring technology described above we have that

$$p^H\left(\theta \middle| S^N, S^M\right) = \begin{cases} 1 & \text{if } \sum_{p \in P} \phi^{s^p} < K \\ \begin{cases} 1 & \text{if } \theta \geq \tilde{\theta} \\ 0 & \text{if } \theta < \tilde{\theta} \end{cases} & \text{if } \sum_{p \in P} \phi^{s^p} \geq K \end{cases} \tag{3.7}$$

where $\tilde{\theta}$ is such that

$$\sum_{p \in P} \mu^p \int_{\tilde{\theta}}^1 \mathbb{1}_{\{\Theta_H^{s^p}\}} f(\theta) \mathrm{d}\theta = K \tag{3.8}$$

Notice that the first row in the right hand side of (3.7) describes the case in which the strategy profiles S^N and S^M are such that the workers competing for a job in H are less than K: the hiring technology will thus hire any worker competing for a job, regardless of her type. If instead a mass of workers greater then K is competing for a job in H, the hiring technology will hire from the most skilled worker downward until the K vacancies are filled: formally it fixes the threshold $\tilde{\theta}$ such that every worker with a type $\theta < \tilde{\theta}$ will have a zero probability of being hired in H. In this context - when there are not enough vacancies for all the workers competing in H - it is useful to denote with $\tilde{\Theta}_H^{s^p, s^{-p}} := \Theta_H^{s^p} \cap [\tilde{\theta}, 1]$ the set of types θ such that $s^p(\theta) = H$ and $p^H(\theta | S^N, S^M) = 1$. For any population p, the mass of workers belonging to $\tilde{\Theta}_H^{s^p, s^{-p}}$ will be denoted by

$$\tilde{\phi}^{s^p} = \mu^p \int_0^1 \mathbb{1}_{\{\tilde{\Theta}_H^{s^p, s^{-p}}\}} f(\theta) \mathrm{d}\theta \tag{3.9}$$

We can now write the expression for the average ability of the workers hired in sector H, that is

$$\theta^H \big(S^N, S^M \big) = \begin{cases} \frac{1}{\sum_{p \in P} \phi^{s^p}} \sum_{p \in P} \phi^{s^p} E\big[\theta \big| \theta \in \Theta_H^{s^p}\big] & \text{if } \sum_{p \in P} \phi^{s^p} < K \\ \frac{1}{\sum_{p \in P} \tilde{\phi}^{s^p}} \sum_{p \in P} \tilde{\phi}^{s^p} E\big[\theta \big| \theta \in \tilde{\Theta}_H^{s^p, s^{-p}}\big] & \text{if } \sum_{p \in P} \phi^{s^p} \geq K \end{cases} \tag{3.10}$$

Let us start the analysis of the pure strategy Nash equilibria of the game through the following result:

Lemma 1 *If the strategy profile (S_*^N, S_*^M) is a pure strategy Nash equilibrium in the labor market, then it is an equilibrium in threshold strategies of the kind (θ_*^N, θ_*^M) such that*

$$s_*^p(\theta) = \begin{cases} H & \textit{if } \theta \geq \theta_*^p \\ L & \textit{if } \theta < \theta_*^p \end{cases} \qquad \forall \ p \in P \tag{3.11}$$

Proof. See Appendix A.

The characterization of pure strategy Nash equilibria of the labor market is given by the next result. For the sake of simplicity we characterize the possible set of equilibria assuming the following restrictions on the payoffs' parameters:

A1. $w(0, \frac{1}{2}) \geq w^L$

A2. $w(1, \frac{1}{2}) - c(0) \leq w^L$

The first assumption implies that, in an economy without migrants, the less skilled native worker prefers to compete in H (notice that she will be hired in H for sure). The second assumption instead restricts the analysis to those cases in which, given zero integration and all the native population competing for a job in H, the most skilled migrant prefers to compete in L (even if she has probability 1 of being hired in H).

Lemma 2 *The profile of thresholds* (θ_*^N, θ_*^M) *such that*

$$s_*^p(\theta) = \begin{cases} H & \text{if } \theta \geq \theta_*^p \\ L & \text{if } \theta < \theta_*^p \end{cases} \qquad \forall\ p \in P$$

is a pure strategy Nash equilibrium in the labor market if and only if it belongs to the set

$$\left\{ (\theta^N, \theta^M) \mid \left(\theta^N = -\frac{\mu^M}{\mu^N}\theta^M + \frac{1-K}{\mu^N} \wedge \theta^M \in \left[1-K, \frac{1-K}{\mu^M}\right] \right) \vee \left(\theta^N = 0 \wedge \theta^M \in \left[\frac{1-K}{\mu^M}, 1\right] \right) \right\}$$

Proof. See Appendix A.

The solid line in Figure 2 represents the set of all potential pure strategy Nash equilibria. At this stage nothing prevents us to think that, for some values of the parameters I, w^L, K and for some specifications of $w(\cdot,\cdot)$ and $c(\cdot)$, we will get multiple equilibria or no equilibrium at all.

We are interested in the behavior of the equilibria for any possible value of I in the policy space $[0, 1]$. In order to get the next result we have to put some more structure on the function $w(\cdot,\cdot)$: in particular, let us assume that, for any potential equilibrium (θ^N, θ^M)

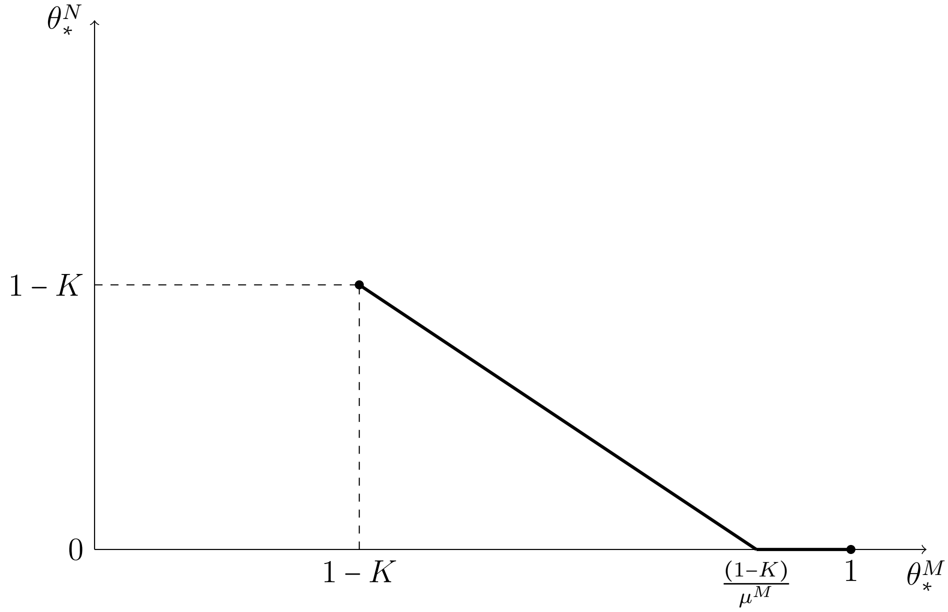

Figure 2: Set of potential Nash equilibria in pure strategies

where $\theta^N = 0$ the following inequality is verified

$$\frac{\partial w(\theta^M, \theta^H)/\partial \theta^M}{\partial w(\theta^M, \theta^H)/\partial \theta^H} > -\frac{d\theta^H(\theta^M)}{d\theta^M} \tag{3.12}$$

Mathematically, this condition requires the total derivative of $w(\cdot,\cdot)$ with respect to individual ability to be positive when computed at the migrants' equilibrium ability threshold.[4] Given this regularity condition we ca state the following

Proposition 1 *For any fixed value of $I \in [0,1]$, the equilibrium (θ^N_*, θ^M_*) exists and it is unique. Moreover, θ^M_* as a function of I is continuous and non increasing (θ^N_* is non decreasing in I).*

Proof. See Appendix A.

The upper diagram in Figure 3 shows the qualitative behavior of the equilibrium thresholds as functions of the policy variable I. The lower diagram shows instead the equilibrium mass of natives workers (red curve), immigrants workers (blue curve) as well as of all workers (black curve) employed in sector H for any value of I.

[4]Notice that, in equilibrium, the function $\theta^H(\cdot,\cdot)$ can be expressed as a function of the migrants' threshold strategy - θ^M - alone since the natives' one - θ^N - can be univocally derived from it.

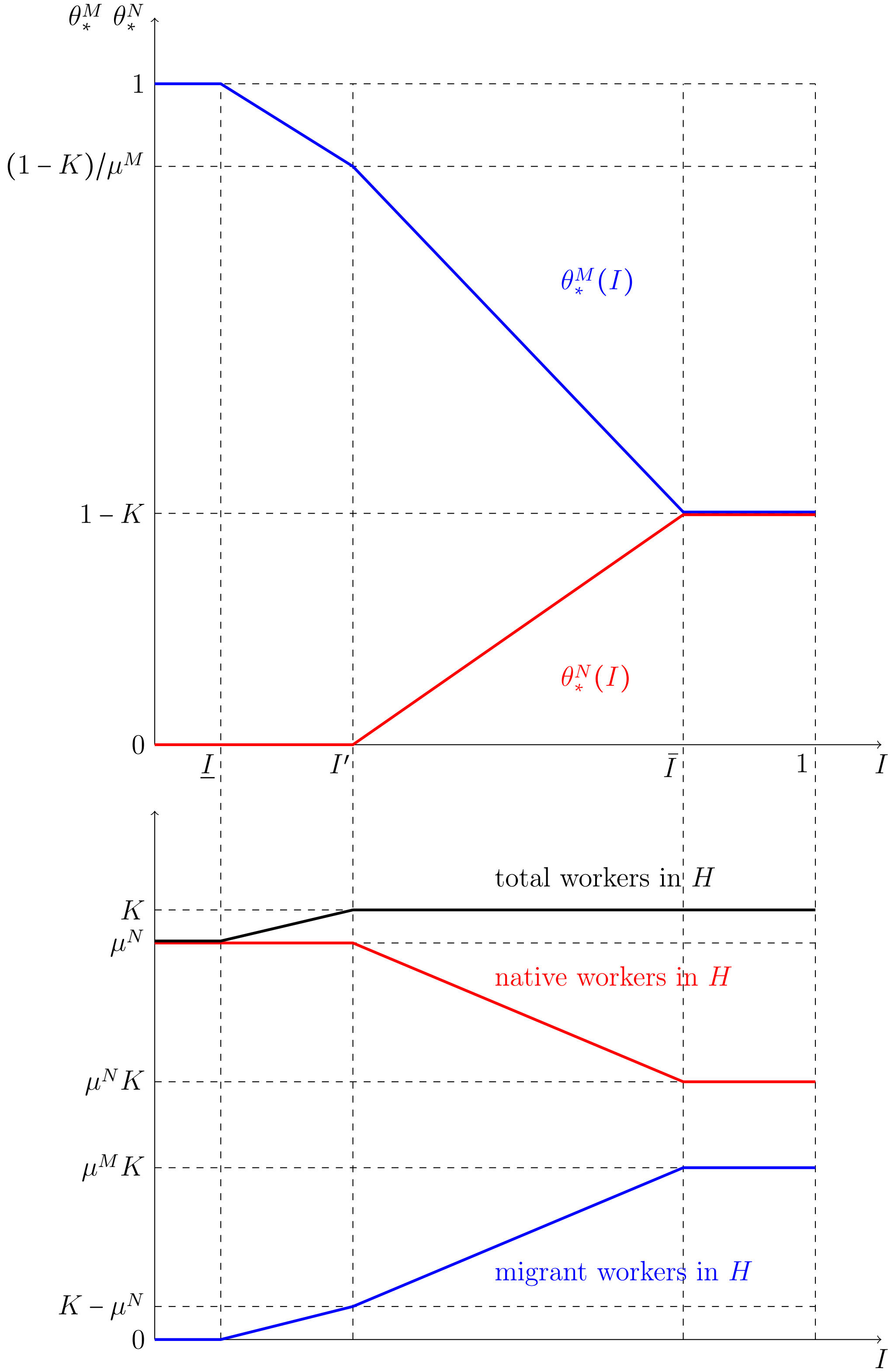

Figure 3: Qualitative behavior of the equilibrium thresholds and of the worker composition in sector H as functions of the level of integration I

When the level of integration is low enough, even the most skilled migrant prefers to get a job in sector L. In this case the equilibrium thresholds are 0 for the natives and 1 for migrants meaning that all the native population competes and is hired in H while all the migrants get employed in L. Therefore in sector H there will be a mass μ^N of native workers, no migrants and a mass $K - \mu^N$ of unfilled vacancies.

Consider now the level of integration $\underline{I}$ such that the most skilled migrant worker is indifferent between H and L. Above $\underline{I}$ there will be an increasing mass of migrants from the right side of the skill distribution willing to compete for a job in H. Until the $K - \mu^N$ vacancies are not filled completely, the migrants competing will be hired in H for sure and the equilibrium thresholds will be 0 for the natives and some continuous and decreasing function of integration (starting from 1) for migrants. The mass of natives in H will be still μ^N while the mass of migrants hired in H will start increasing with I.

At the level of integration for which the mass of competing migrants fills all the $K - \mu^N$ vacancies (we call this level of integration I'), the indifferent migrant has type $\frac{1-K}{\mu^M}$. Above I', for any further mass of migrants hired in H there will be an analogous mass of natives from the left side of the skill distribution that will have 0 probability of being hired (their ability is lower than the one of the new migrants willing to compete). In this case the equilibrium threshold for the natives starts to depart from 0 and the one for migrants continues to decrease below $\frac{1-K}{\mu^M}$. What happens in H for levels of integration above I'? The equilibrium mass of native workers hired in the sector starts to decrease while the mass of migrants continues to increase with I. This commovement is such that the K vacancies are always filled completely.

Finally, consider the level of integration $\bar{I}$ such that the ability type of the indifferent migrant is identical to the ability type of the less skilled native employed in H. In equilibrium this type is $1 - K$. For any value of $I \geq \bar{I}$ the probability of being hired in H will be 0 for any worker with a type $\theta < 1 - K$. Therefore above $\bar{I}$ the equilibrium thresholds will stay constant and equal to $1 - K$. The mass of natives employed in H reaches its minimum level of $\mu^N K$ and the mass of migrants its maximum of $\mu^M K$.

3.2 Political economy equilibrium

In this section we consider a voting procedure on the policy variable I. Before the workers's decision, natives agents vote and the level of integration is implemented accordingly to the voting outcome. A median voter political economy equilibrium aggregates the natives' preferences over I. The timing of the economy is represented in Figure 4.

Figure 4: Timing of the economy

For any $\theta \in [0,1]$ we want to determine the preferred integration policy of a type θ native agent. Intuitively, the trade-off that she faces is the following: a high level of integration might let high skill migrants enter sector H, enhancing the average productivity in the sector and, in turns, increasing the salary of any other worker in H. On the other hand, high integration might force some natives to compete for a job in L, making impossible for them to be hired in H. The preferred integration policy of a type θ native is the one that maximizes the average ability in sector H, provided that the equilibrium threshold is such that the type θ native gets a job in H. Formally, the preferred policy of a type θ native solves the following maximisation problem

$$\max_{I \in [0,1]} \theta^H(\theta^M_*(I)) \quad \text{s.t.} \quad \theta^N_*(I) \leq \theta \tag{3.13}$$

The mapping from the voter's type to the solution(s) of problem (3.13) can have different forms depending on the values of the parameters K and μ^M.[5] The possible cases depend upon the shape of the function $\theta^H(\theta^M)$ and its local and global maxima.

In particular we can distinguish two relevant cases. In the first one, the average ability in sector H is globally maximised hiring more able migrants instead of less able natives

[5] Given our normalization assumption $\mu^N + \mu^N = 1$ the only relevant demographic parameter is the relative mass of migrants $\mu^M/(1-\mu^M)$, which is strictly monotone, increasing function of μ^M.

until the K vacancies are completely filled (the argmax being $\theta^M = 1 - K$). This happens either when the K vacancies are not too many or if the relative mass of migrants is not too big. If instead there is a big number of vacancies or the relative mass of migrants is high enough, the global maximum for θ^H is reached at $\theta^M \geq (1 - K)/\mu^M$. Under this second case all the natives get a position in H when θ^H is at its maximum.

Consider the first case (θ^H has a unique global maximum for $\theta^M = 1 - K$). A native worker with type $\theta < 1 - K$ would solve problem (3.13) choosing I_* such that $\theta_*^N(I_*) = \theta$. Intuitively, she would prefer a level of integration as high as possible under the constraint that the probability of being hired in H the next period is equal to 1. Notice that for a voter of type $\theta = 0$ the preferred level of integration will be the highest one delivering the equilibrium threshold $\theta_*^N = 0$, i.e. I'. On the contrary, a native of type $\theta \geq 1 - K$ will never run the risk of loosing her potential job position in H: any level of integration that gives the $\theta^M = 1 - K$ in equilibrium would solve problem (3.13). In particular this is true for any value of I in the set $[\bar{I}, 1]$. From Proposition 1 we know that θ_*^N is non decreasing in I (more precisely, it is strictly increasing on $[I', \bar{I}]$). Therefore for any type $\theta \in [0, 1 - K)$ the preferred policy of the voter will be an increasing function of her ability type. In the second case - where $\theta^H(\theta^M)$ has a unique global maximum at $\theta^M \geq (1 - K)/\mu^M$ - for any ability type θ the constraint of problem (3.13) will not be binding at the optimum. Therefore, every native worker prefers the policy that maximizes θ^H.

The following result characterizes the political economy equilibrium under classical Downsian electoral competition[6]

Proposition 2 *the political economy equilibrium I_* always exists. Moreover, If the parameters μ^M and K are such that the following condition holds*

$$\mu^M K < 2\sqrt{1 - \mu^M} - 2(1 - \mu^M) \tag{3.14}$$

[6]Classical Downsian electoral competition consists in the following. Two candidates maximize the probability of winning. First, electoral platforms are announced. Second, majority voting aggregates the preferences of the native workers. Finally, the winner's platform is implemented assuming perfect commitment.

I_* can take any value in $[\bar{I}, 1]$, where

$$\bar{I} := c^{-1}\Big[w\Big(1-K, 1-\frac{K}{2}\Big) - w^L\Big] \tag{3.15}$$

Otherwise, $I_* = I''$, where

$$I'' := c^{-1}\Big[w\Big(\frac{1-\sqrt{1-\mu^M}}{\mu^M}, \frac{1-\sqrt{1-\mu^M}}{\mu^M}\Big) - w^L\Big] < \bar{I} \tag{3.16}$$

Proof. See Appendix A.

The equilibrium described in Proposition 2 reflects the preferences of the median voter. The standard uniform distribution of abilities and our assumptions on the parameters imply that the median voter has an ability type $\theta = 1/2 > 1-K$. Within this simple framework the median voter is always hired with probability one in sector H. Therefore, the following results have to be interpreted in a context where at least half of the voting population does not suffer an economic loss due to migrants' participation in the labor market.[7] The natives workers which might suffer from migrants' participation are those on the left tail of the ability distribution. This is in line with a general empirical finding that the negative effects of migration (if any) are concentrated asymmetrically on the low-skilled (or low-wages) native workers (see for instance Borjas, 2003; Dustmann et al., 2005; Ottaviano and Peri, 2012; Dustmann et al., 2013). Our model implies that the preferred policy of those native workers (from a 0 ability type to $\theta = 1-K$) might be a lower level of integration. Intuitively, this is the case when the number of vacancies is sufficiently low. Notice that the constraint on K is less tight when the relative stock of migrants is small. When there is enough excess labor demand in sector H (above the size of the native population), no native worker is risking her position in H and the optimal integration policy is the same for every ability type.

[7]This restriction is consistent with the recent empirical evidence on the effect of migration for natives' employment and wages. For instance, looking at the labor market in the UK, Dustmann et al. (2013) estimate that only 20% of natives workers suffer some wage reduction because of migration.

3.3 Comparative statics

The lower bound integration in the first case of Proposition 2, $\bar{I}$, is increasing in K and w^L. For the sake of simplicity, we assume that the implemented policy is always the minimum value $\bar{I}$.[8] Instead, the level of integration implemented in the second case, I'', is decreasing in μ^M (increasing in μ^N) and increasing in w^L.

The first interesting finding is that integration is always increasing in w^L. We interpret the L sector as a metaphor for those economic opportunities that a migrant worker has, irrespectively of the degree of integration. These go from the opportunities in the informal economy to those low skill positions for which the native labor supply is shrinking. Concrete examples are jobs in the domestic sector (carers and house keepers), in the construction sector, in agriculture. The better those opportunities are in terms of wages and working conditions the higher the opportunity cost for a migrant to compete for a better position (a job in sector H); therefore the level of integration needed to attract the most skilled migrants to the jobs that have positive externalities for the natives should be higher as well.

Secondly, when the average ability in the H sector is maximized not hiring those workers (natives and migrants) with an ability type smaller than $1 - K$, the exogenous mass of vacancies K is relevant in determining the equilibrium outcome. In particular we have that integration is increasing in K. More job positions in H mean the possibility for any type θ voter to be keen to attract - ceteris paribus - more migrants in the sector. This translates into a higher integration in equilibrium. In our reduced form labor market, we can interpret a higher value of K as a higher employment capacity of the economy. Employment capacity matters for the equilibrium value of integration when it is small enough. In particular, this constraint is less tight the lower the relative stock of migrants in the economy (see condition (3.14) in Proposition 2). In those environments higher employment capacity increases the benefits for the natives of migrants's participation in sector H and allows for higher values of integration in equilibrium.

[8] In order to justify this assumption we can think of some strictly increasing implementation costs of integration.

Finally, the model tells us that, when the average ability in H is maximized keeping all native workers in the sector, the demographic composition of society becomes relevant. In particular, the equilibrium value of integration is increasing in the relative share of migrants μ^M. Given the uniform distribution of abilities, the higher the number of migrants, the higher the positive externalities that natives agents can get from integration (formally, the maximum of $\theta^H(\theta^M)$ is increasing in μ^M).

Simple descriptive evidence from the MIPEX host economies, suggests that the role of K is actually predominant over the role of μ^M. Figure 5 plots the MIPEX Labor market mobility indicator against the employment to population ratio, taken as a proxy for employment capacity in the labor market.[9]

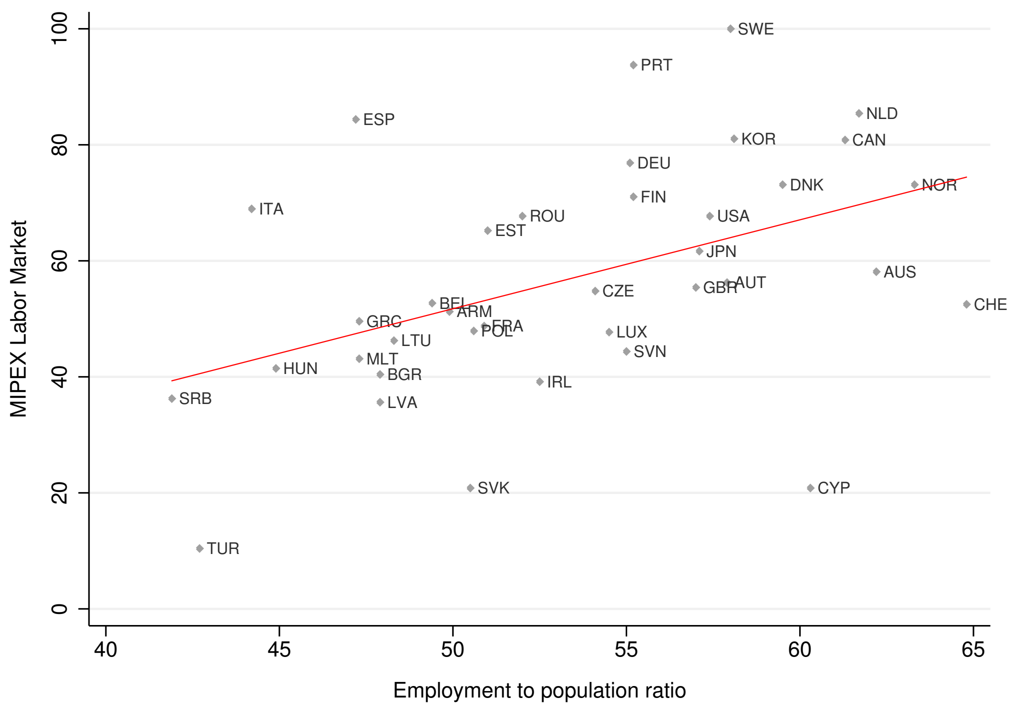

Figure 5: Migrants' integration and employment capacity in 2010

As predicted by our model, the linear fit (red line in Figure 5) is upward sloped with statistical significance (slope coefficient equal to +1.533; p-value 0.008). The analogous positive co-movement between integration and the share of migrants is absent from our

[9]Employment to population ratio is the proportion of a country's population that is employed. Ages 15 and older are generally considered the working-age population. Employment to population ratio data come from the International Labour Organisation.

data. Applying our theoretical mechanism to the data and taking this suggestive empirical evidence with the necessary degree of caution, it seems that the employment capacity of the economies in our sample is low enough to be a relevant determinant of integration. A rigorous empirical exercise to test the model's implications goes beyond the scope of this paper and it is left for further research.

4 Conclusions

This paper addresses the issue of migrants' integration into a host country labor market. As stated in a recent research report by the OECD and the European Union "the integration of immigrants and their children [is] high on the policy agenda of EU and OECD countries, both from an economic and a social standpoint. The active participation of immigrants and their children in the labour market and, more generally, in public life is vital for ensuring social cohesion in the host country" (OECD/EU, 2015, pg. 9). A critical feature of many host economies is the asymmetry in labor market opportunities between migrant and native born workers. The focus of the paper is on those integration policies that enhance migrants' capacity to access the labor market. We model a simple trade-off embedded in integration policies. On the one hand they allow the best migrants to access the labor market with positive spillovers for the native workers through higher productivity. On the other, high skilled migrant's access might reduce the probability of less skilled native workers to be hired in high productivity sectors.

Our model solves for the integration policy implemented in a political economy where only native workers vote. When the employment capacity of the host country is not too high, the equilibrium requires that some low-skill native agents are not hired in the high productivity sector in order to let more qualified migrants to enter. When this is the case, the level of integration is increasing in the employment capacity of the country. We find rough empirical evidence in support of this result using data from the MIPEX project. Moreover, our model predicts that the level of integration will be a positive function of the opportunities in the informal economy. The higher such opportunities, the more the high-skill migrants tend to avoid positions in the formal market for which they are in a disadvantaged position with respect to native born workers. In order to attract the high skill migrants integration has to be higher.

This paper contributes to the economic literature on migration. It does so introducing a working definition of labor market integration and setting a simple political economy framework to study its determinants given the relevant economic trade-off between the positive spillovers of migrants' participation and its potential negative effect on the low-skill native workers. Our analysis leaves some interesting questions open for future research. On the one hand, relaxing the assumptions of Downsian electoral competition and native voting, would allow us to study the role of special interest groups, ideological preferences as well as migrant voting in determining the level of integration above and beyond the role of modelled economic trade-off. On the other, it is important to bring our theoretical implications to the data, carefully controlling for non economic determinants of integration.

References

Benhabib, J., "On the political economy of immigration," *European Economic Review*, 1996, *40* (9), 1737–1743.

Borjas, George J., "The Labor Demand Curve Is Downward Sloping: Reexamining the Impact of Immigration on the Labor Market," *The Quarterly Journal of Economics*, 2003, *118* (4), 1335–1374.

Dolmas, J. and G. W. Huffman, "On the political economy of immigration and income redistribution," Vanderbilt University Working Paper May 2003.

Dustmann, C., T. Frattini, and I. Preston, "The effect of immigration along the distribution of wages," *Review of Economic Studies*, 2013, *80* (1), 145–173.

Dustmann, Christian, Francesca Fabbri, and Ian Preston, "The Impact of Immigration on the British Labour Market," *The Economic Journal*, 2005, *115* (507), F324–F341.

Eguia, J. X., "A Theory of Discrimination and Assimilation," Unpublished manuscript 2011.

Facchini, G. and A. M. Mayda, "From individual attitudes towards migrants to migration policy outcomes: Theory and evidence," *Economic Policy*, 2008, *56*, 651–713.

Goyal, S. and F. Vega-Redondo, "Network formation and social coordination," *Games and Economic Behavior*, 2005, *50* (2), 178–207.

OECD/EU, *Indicators of Immigrant Integration 2015: Settling In*, Paris: OECD, 2015.

Ortega, F., "Immigration quotas and skill upgrading," *Journal of Public Economics*, 2005, *89* (9-10), 1841–1863.

Ottaviano, Gianmarco I. P. and Giovanni Peri, "Rethinking the Effect of Immigration on Wages," *Journal of the European Economic Association*, 2012, *10* (1), 152–197.

Reich, B., "Culture, Community and Segregation," Unpublished manuscript, Cambridge University November 2011.

Appendices

A Proofs

Proof of Lemma 1. Assume by way of contradiction that the equilibrium (S_*^N, S_*^M) is not in threshold strategies. In particular assume that there exist two values θ_1 and θ_2, with $\theta_1 < \theta_2$, such that $s_*^M(\theta_1) = s_*^M(\theta_2) = H$ and, for any $\theta \in (\theta_1, \theta_2)$, $s_*^M(\theta) = L$ (the same proof can be done for $p = N$). The equilibrium implies that

$$p^H\Big(\theta_1\Big|S_*^N, S_*^M\Big) = p^H\Big(\theta_2\Big|S_*^N, S_*^M\Big) = 1 \quad \text{and} \tag{A-1}$$

$$w\Big(\theta_1, \theta^H\big(S_*^N, S_*^M\big)\Big) - c(I) \geq w^L \tag{A-2}$$

Notice that $w\Big(\theta_1, \theta^H\big(S_*^N, S_*^M\big)\Big) < w\Big(\theta_2, \theta^H\big(S_*^N, S_*^M\big)\Big)$ by construction and thus

$$w\Big(\theta_2, \theta^H\big(S_*^N, S_*^M\big)\Big) - c(I) > w^L \tag{A-3}$$

Consider now a player i in M with any type $\theta \in (\theta_1, \theta_2)$. Denote $\hat{S}_*^M$ the strategy profile where i deviates to H while all the other players in M play as prescribed by S_*^M. Given that $p^H\Big(\theta_1\Big|S_*^N, S_*^M\Big) = 1$ and $\theta > \theta_1$ we have that

$$p^H\Big(\theta\Big|S_*^N, S_*^M\Big) = 1 \tag{A-4}$$

Moreover,given that $w(\cdot,\cdot)$ is increasing in the first argument, (A-2) implies

$$w\Big(\theta, \theta^H\big(S_*^N, \hat{S}_*^M\big)\Big) - c(I) > w^L \tag{A-5}$$

Given (A-4) and (A-5), it is immediate to see that it is profitable for a type θ choose H instead of L, which contradicts our assumption of equilibrium.

The same logic can be used to show a contradiction in the case of threshold strategies of the kind

$$s_*^p(\theta) = \begin{cases} L & \text{if } \theta \geq \theta_*^p \\ H & \text{if } \theta < \theta_*^p \end{cases} \qquad \forall\ p \in P$$

This observation completes the proof. ■

Proof of Lemma 2.

Alternative statement, preliminary notation and derivations First, notice that Lemma 2 can be rewritten as follows

Lemma 3 *Consider the following system in θ^N and θ^M:*

$$\begin{cases} w\Big(\theta^M, \theta^H\big(\theta^N, \theta^M\big)\Big) - c(I) - w^L = 0 \\ \theta^N = \begin{cases} 0 & \textit{if } \theta^M \in \left(\frac{1-K}{\mu^M}, +\infty\right) \\ -\frac{\mu^M}{\mu^N}\theta^M + \frac{1-K}{\mu^N} & \textit{if } \theta^M \in \left[1-K, \frac{1-K}{\mu^M}\right] \\ 1-K & \textit{if } \theta^M \in [0, 1-K) \end{cases} \end{cases} \tag{A-6}$$

The profile of thresholds (θ^N_*, θ^M_*) such that

$$s^p_*(\theta) = \begin{cases} H & \textit{if } \theta \geq \theta^p_* \\ L & \textit{if } \theta < \theta^p_* \end{cases} \quad \forall\ p \in P$$

is a pure strategy Nash equilibrium in the labor market if and only if one of the following conditions is true:

- *(θ^N_*, θ^M_*) solves (A-6) and it belongs to the set $[0, 1-K] \times [1-K, 1]$,*
- *$(\theta^N_*, \theta^M_*) = (1-K, 1-K)$ if at least one solution of (A-6) belongs to the set $\{1-K\} \times [0, 1-K)$,*
- *$(\theta^N_*, \theta^M_*) = (0, 1)$ if at least one solution of (A-6) belongs to the set $\{0\} \times [1-K, +\infty)$.*

By the result of Lemma (1) we can rewrite the mass of workers in population p that will play H according to some threshold strategy θ^p as

$$\phi^{\theta^p} = \mu^p\big(1 - \theta^p\big) \tag{A-7}$$

Let us define the the function $\tilde{F}^H\big(\theta|\theta^N, \theta^M\big)$ that, given the profile of threshold strategies, for any value of θ gives the total (from both populations) mass of workers competing in H with a type smaller or equal to θ. We can write the expression of $\tilde{F}^H$ as follows

$$\tilde{F}^H\big(\theta|\theta^N, \theta^M\big) = \begin{cases} 0 & \text{if } \theta < \min\{\theta^N, \theta^M\} \\ \sum_{p \in P} \mathbb{1}_{\{\theta^p < \theta^{-p}\}} \mu^p(\theta - \theta^p) & \text{if } \min\{\theta^N, \theta^M\} \leq \theta < \max\{\theta^N, \theta^M\} \\ \theta - \sum_{p \in P} \mu^p \theta^p & \text{if } \max\{\theta^N, \theta^M\} \leq \theta \leq 1 \end{cases} \tag{A-8}$$

It is useful to define the function $G^H\big(\theta|\theta^N, \theta^M\big)$ that, given the profile of threshold strategies, for any value of θ gives the total mass of workers competing in H with a type bigger or equal to θ. We can write the expression of G^H as follows:

$$G^H\big(\theta|\theta^N, \theta^M\big) = \begin{cases} 1 - \sum_{p \in P} \mu^p \theta^p & \text{if } \theta < \min\{\theta^N, \theta^M\} \\ \sum_{p \in P} \mathbb{1}_{\{\theta^p < \theta^{-p}\}} \Big[\mu^p(1-\theta) + \mu^{-p}(1 - \theta^{-p})\Big] & \text{if } \min\{\theta^N, \theta^M\} \leq \theta < \max\{\theta^N, \theta^M\} \\ 1 - \theta & \text{if } \max\{\theta^N, \theta^M\} \leq \theta \leq 1 \end{cases} \tag{A-9}$$

Now we can rewrite the expressions of $p^H(\cdot|\cdot, \cdot)$ and $\theta^H(\cdot, \cdot)$ as

$$p^H\big(\theta\big|\theta^N,\theta^M\big)=\begin{cases}\begin{cases}1 & \text{if } \theta\geq\min\{\theta^N,\theta^M\}\\ 0 & \text{if } \theta<\min\{\theta^N,\theta^M\}\end{cases} & \text{if } G\big(\min\{\theta^N,\theta^M\}\big|\theta^N,\theta^M\big)\leq K\\ \begin{cases}1 & \text{if } \theta\geq\hat{\theta}\\ 0 & \text{if } \theta<\hat{\theta}\end{cases} & \text{if } G\big(\min\{\theta^N,\theta^M\}\big|\theta^N,\theta^M\big)>K\end{cases} \tag{A-10}$$

$$\theta^H\big(\theta^N,\theta^M\big)=\begin{cases}\frac{1}{\sum_{p\in P}\phi^{\theta^p}}\sum_{p\in P}\phi^{\theta^p}E\big[\theta\big|\theta\geq\theta^p\big] & \text{if } G\big(\min\{\theta^N,\theta^M\}\big|\theta^N,\theta^M\big)\leq K\\ \frac{1}{\sum_{p\in P}\tilde{\phi}^{\theta^p}}\sum_{p\in P}\tilde{\phi}^{\theta^p}E\big[\theta\big|\theta\geq\max\{\theta^p,\hat{\theta}\}\big] & \text{if } G\big(\min\{\theta^N,\theta^M\}\big|\theta^N,\theta^M\big)>K\end{cases} \tag{A-11}$$

where $\hat{\theta}$ is such that

$$G\big(\hat{\theta}\big|\theta^N,\theta^M\big)=K \tag{A-12}$$

and $\tilde{\phi}^{\theta^p}$ is defined as follows

$$\tilde{\phi}^{\theta^p}:=\mu^p\big(1-\max\{\theta^p,\hat{\theta}\}\big) \tag{A-13}$$

Body of the proof For the workers in the native population optimality requires that

$$\theta^N_*=\min\Big\{\theta\in[0,1]:\ w\Big(\theta,\theta^H\big(\theta,\theta^M_*\big)\Big)p^H\big(\theta\big|\theta,\theta^M_*\big)\geq w^L\Big\} \tag{A-14}$$

Looking at the workers in the migrant population it is convenient to define the following sets:

$$Q:=\Big\{\theta\in[0,+\infty):\ w\Big(\theta,\theta^H\big(\theta^N_*,\theta\big)\Big)-c(I)-w^L=0\Big\} \tag{A-15}$$

$$B:=\Big\{\theta\in[0,+\infty):\ p^H\big(\theta\big|\theta^N_*,\theta\big)=1\Big\} \tag{A-16}$$

Optimality requires that:

$$\theta^M_*\in\begin{cases}\{1\} & \text{if } Q\cap B\neq\varnothing\ \wedge\ Q\cap B\cap[0,1]=\varnothing\\ Q\cap B\cap[0,1] & \text{if } Q\cap B\cap[0,1]\neq\varnothing\\ \{\min B\} & \text{if } Q\cap B=\varnothing\end{cases} \tag{A-17}$$

Notice that the first row in (A-17) is not logically correct: if the solution of the indifference condition is above one, the type 1 migrant worker will always choose L. With our notation we are saying that she is always indifferent. This approach reduces the number of cases to consider and does not create any algebraic problem given the continuity of our populations. To solve the logical problem it is sufficient to assume that, whenever $\theta^M_*=1$, a type 1 indifferent migrant worker will choose H only if 1 is a solution of the indifference condition; otherwise the indifferent migrant will always choose L. Another way to get rid of this problem is to assume that $w(1,\frac{1}{2})-c(0)=w^L$.

By definition, the profile of thresholds (θ_*^N, θ_*^M) such that

$$s_*^p(\theta) = \begin{cases} H & \text{if } \theta \geq \theta_*^p \\ L & \text{if } \theta < \theta_*^p \end{cases} \qquad \forall\ p \in P$$

is a pure strategy Nash equilibrium in the labor market if and only if it solves jointly (A-14) and (A-17).

Notice that (A-14) is equivalent to

$$\theta_*^N = \min\Big\{\theta \in [0,1]:\ p^H\big(\theta\big|\theta, \theta_*^M\big) = 1\Big\} \tag{A-18}$$

This equivalence is given by the following facts:

- $w(0, \frac{1}{2}) \geq w^L$,
- $\frac{1}{2} = \min_{(\theta^N, \theta^M) \in [0,1]^2} \theta^H\big(\theta^N, \theta^M\big)$
- $w(\cdot, \cdot)$ is increasing in both arguments.

Given the expression of $p^H\big(\theta\big|\theta^N, \theta^M\big)$ we have that (A-18) is equivalent to

$$\theta_*^N = \min\Big\{\theta \in [0,1]:\ G\big(\theta\big|\theta, \theta_*^M\big) \leq K\Big\} \tag{A-19}$$

Using the expression of $G^H\big(\theta|\theta^N, \theta^M\big)$ we can rewrite (A-18) in the following way:

$$\theta_*^N = \begin{cases} 0 & \forall\ \theta_*^M \text{ such that } G\big(0\big|0, \theta_*^M\big) < K \\ -\frac{\mu^M}{\mu^N}\theta_*^M + \frac{1-K}{\mu^N} & \forall\ \theta_*^M \text{ such that } G\big(0\big|0, \theta_*^M\big) \geq K \text{ and } \theta_*^M > -\frac{\mu^M}{\mu^N}\theta_*^M + \frac{1-K}{\mu^N} \\ 1-K & \forall\ \theta_*^M \text{ such that } G\big(0\big|0, \theta_*^M\big) \geq K \text{ and } \theta_*^M \leq -\frac{\mu^M}{\mu^N}\theta_*^M + \frac{1-K}{\mu^N} \end{cases} \tag{A-20}$$

Notice that (A-18) and (A-17) imply $\theta_*^N \leq \theta_*^M$, therefore (A-20) becomes

$$\theta_*^N = \begin{cases} 0 & \forall\ \theta_*^M \text{ such that } \mu^N + \mu^M(1-\theta_*^M) < K \\ -\frac{\mu^M}{\mu^N}\theta_*^M + \frac{1-K}{\mu^N} & \forall\ \theta_*^M \text{ such that } \mu^N + \mu^M(1-\theta_*^M) \geq K \text{ and } \theta_*^M > 1-K \\ 1-K & \forall\ \theta_*^M \text{ such that } \mu^N + \mu^M(1-\theta_*^M) \geq K \text{ and } \theta_*^M = 1-K \end{cases} \tag{A-21}$$

which can be rewritten as

$$\theta_*^N = \begin{cases} 0 & \text{if } \theta_*^M \in (\frac{1-K}{\mu^M}, 1] \\ -\frac{\mu^M}{\mu^N}\theta_*^M + \frac{1-K}{\mu^N} & \text{if } \theta_*^M \in [1-K, \frac{1-K}{\mu^M}] \end{cases} \tag{A-22}$$

Let us look at the migrant worker's condition. We can rewrite the set B as

$$B = \Big\{\theta \in [0, +\infty):\ \theta \geq \theta_*^N\Big\} \tag{A-23}$$

and, given (A-22) we have that $B = [1 - K, +\infty)$. Therefore (A-17) becomes

$$\theta_*^M \in \begin{cases} \{1\} & \text{if } Q \cap [1-K,+\infty) \neq \varnothing \ \wedge \ Q \cap [1-K,1] = \varnothing \\ Q \cap [1-K,1] & \text{if } Q \cap [1-K,1] \neq \varnothing \\ 1-K & \text{if } Q \cap [1-K,+\infty) = \varnothing \end{cases} \tag{A-24}$$

We can put together (A-24) and (A-22) and we get that

- for any $\theta \in Q \cap (1,+\infty)$, $(\theta_*^N, \theta_*^M) = (0,1)$
- for any $\theta \in Q \cap [1-K,1]$, $\theta_*^M = \theta$ and θ_*^N is given by (A-22)
- for any $\theta \in Q \cap [0,1-K)$, $(\theta_*^N, \theta_*^M) = (1-K, 1-K)$.

■

Proof of Proposition 1. We can first use the results in Lemma (1) and Lemma (2) to derive an expression of the equilibrium value of θ^H as a function of the equilibrium value of θ^M:

$$\theta^H(\theta^M) = \begin{cases} \frac{1-\mu^M\theta^M}{\mu^N} - \frac{K}{2\mu^N} - \frac{\mu^M}{2K\mu^N}(\theta^M-1)^2 & \text{if } \theta^M \in [1-K, \frac{1-K}{\mu^M}] \\ \frac{1-\mu^M(\theta^M)^2}{2(1-\mu^M\theta^M)} & \text{if } \theta^M \in (\frac{1-K}{\mu^M}, 1] \end{cases} \tag{A-25}$$

We can rewrite the first equation of (A-6) using (A-25) as

$$w\Big(\theta^M, \theta^H\big(\theta^M\big)\Big) - c(I) - w^L = 0 \tag{A-26}$$

Consider the left hand side of (A-26) as a function $g\big(I, \theta^M\big)$ where $g : [0,1]^2 \longrightarrow \mathbb{R}$

By the theorem of the global existence of the implicit function we have that there exists a unique function $\psi : D \subset [0,1] \longrightarrow \mathbb{R}$ such that $g\big(I, \psi(I)\big) = 0 \quad \forall\ I \in D$; ψ is continuous and takes values in $[1-K, 1]$ while $D = [\underline{\underline{I}}, \bar{I}]$ where

$$\underline{\underline{I}} = c^{-1}\Big[w\Big(1, \frac{1}{2}\Big) - w^L\Big] \text{ and } \bar{I} = c^{-1}\Big[w\Big(1-K, 1-\frac{K}{2}\Big) - w^L\Big] \tag{A-27}$$

Indeed the assumptions of the theorem of global existence of the implicit function are satisfied on the set $D \times [1-K, 1]$. More precisely,

- g is continuous in $D \times [1-K, 1]$;
- in $D \times [1-K, 1]$, g is strictly monotone with respect to θ^M for any fixed I;
- for any fixed $I \in D$, the function g changes sign varying θ^M in $[1-K, 1]$.

We have shown existence and uniqueness of the equilibrium for $I \in D$. For any value of $I \in (\bar{I}, 1]$ it is easy to see that the equilibrium exists unique and it is equal to $(1-K, 1-K)$. Instead, for any value of $I \in [0, \underline{\underline{I}})$ we always have the unique equilibrium $(0,1)$.

To show that θ_*^M is non increasing in I we start applying the implicit function theorem. In particular, for any point $(I_0, \theta_0^M) \in D \times [1-K, 1]$ we have that

- g is C^1 in a neighborhood $V(I_0, \theta_0^M)$ and
- $\frac{\partial g}{\partial \theta^M}(I_0, \theta_0^M) \neq 0$.

Then, by the IFT, there exists a neighborhood $U(I_0)$ such that

$$\psi'(I) = -\frac{\frac{\partial g}{\partial I}(I, \psi(I))}{\frac{\partial g}{\partial \theta^M}(I, \psi(I))} = -\frac{c'(I)}{\frac{\partial w(\theta^M,\theta^H)}{\partial \theta^M} + \frac{\partial w(\theta^M,\theta^H)}{\partial \theta^H}\frac{d\theta^H(\theta^M)}{d\theta^M}} < 0 \quad \forall\ I \in U(I_0) \tag{A-28}$$

The following facts complete the argument:

- $\forall\ I \in [0, \underline{\underline{I}})$, $\theta_*^M(I) = 1$,
- $\theta_*^M(\underline{\underline{I}}) = \psi(\underline{\underline{I}}) = 1$,
- $\forall\ I \in [\underline{\underline{I}}, \bar{I}]$, $\theta_*^M(I) = \psi(I)$ which is decreasing in I,
- $\theta_*^M(\bar{I}) = \psi(\bar{I}) = 1 - K$ and
- $\forall\ I \in (\bar{I}, 1]$, $\theta_*^M(I) = 1 - K$.

Given the expression of θ_*^N as a function of θ_*^M given by the second equation in (A-6) it is immediate to see that θ_*^N is non decreasing in I. ■

Proof of Proposition 2. The mapping from the voter's type to the solution(s) of problem (3.13) can have different forms depending on the values of the parameters K and μ^M. The possible cases depend upon the shape of the function $\theta^H(\theta^M)$ and its local and global maxima. In particular, taking the expression for $\theta^H(\cdot)$ from equation (A-25) and maximizing it we have the following cases:

1. $\theta^H(\theta^M)$ has a unique maximum $\theta_1^H = 1 - K/2$ at $\theta_1^M = 1 - K$. This is the case if $K < \sqrt{1 - \mu^M}$;
2. $\theta^H(\theta^M)$ has a global maximum at θ_1^M and a local maximum $\theta_2^H = \frac{1-\sqrt{1-\mu^M}}{\mu^M}$ at $\theta_2^M = \frac{1-\sqrt{1-\mu^M}}{\mu^M}$. This is the case if $K > \sqrt{1 - \mu^M}$ and $K < \frac{2\sqrt{1-\mu^M}-2(1-\mu^M)}{\mu^M}$;
3. $\theta^H(\theta^M)$ has a local maximum at θ_1^M and a global maximum at θ_2^M. This is the case if $K > \frac{2\sqrt{1-\mu^M}-2(1-\mu^M)}{\mu^M}$;
4. $\theta^H(\theta^M)$ has two equal maxima at θ_1^M and θ_2^M. This is the case if $K = \frac{2\sqrt{1-\mu^M}-2(1-\mu^M)}{\mu^M}$.

Assuming that when indifferent (case 4.) a voter chooses the lowest level of integration, the result is immediately given from the fact that voters' preferences satisfy Gans-Smart single corossing condition. ■